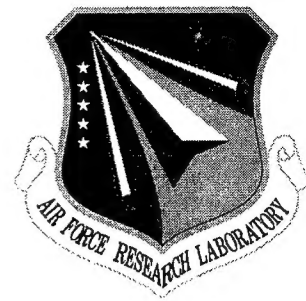AFRL-IF-RS-TR-2001-54
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WAVEFORM AND TRANSCEIVER DESIGNS FOR SMART RADIO

Alan R. Lindsey

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13. ABSTRACT (Maximum 200 words)

Utilizing multidimensional signaling techniques, a generalized multirate wavelet-based modulation format for orthogonally multiplexed communication systems is presented. Wavelet Packet Modulation (WPM) employs the basis functions from an arbitrary pruning of a dyadic tree structured filter bank as orthogonal pulse shapes for conventional Quadrature Amplitude Modulation (QAM) symbols. This generalized framework affords an entire library of basis sets with increased flexibility in time-frequency partitioning. The bandwidth efficiency and power spectral density for the general signal have been analytically derived and it is shown that these figures of merit, for every possible permutation of wavelet packet filter banks, are equivalently that of standard QAM. Hence, WPM is directly applicable in all existing systems employing this modulation format, with the added benefit of arbitrary time-frequency plane configurations. This benefit is not to be understated - flexibility in both time and frequency are extremely important for mitigating certain types of channel impairments, e.g. hostile narrowband and/or impulsive jamming/countermeasures. The fact that, by a simple hardware exchange, this flexibility is afforded for communication links currently in operation is of critical value to DoD-wide communication system designers. The following report details the work accomplished for this project - a survey of relevant work, wavelet packet modulation, the discrete implementation of WPM, and the spread spectrum extension of WPM.

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Wavelet Packet Modulation, Smart Radio, Interference Excision, spread spectrum, jamming, jammer, signal conditioning, immune receivers, time-frequency, waveform

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1 EXECUTIVE SUMMARY

The term "Smart Radio" is intended to capture the essence of a body of work devoted to the development of interoperable, modular, portable, network-aware, multiple-mode radios for military (and now commercial) deployment across all defense services – Army, Navy, Air Force, Marines, Coast Guard, Reserves, etc., and interoperable with force protection activities such as intelligence, reconnaissance, and surveillance equipment and personnel. While the Smart Radio obviously entails numerous design issues such as protocols, architectures, hardware, antennas, digital signal processing, analog electronics, etc., this effort specifically addresses integrated waveform algorithms (in a digital context) and transceiver designs. The goal is to develop waveforms and, by default, transmitters and receivers that generate and demodulate them, which incorporate the constraints of the implementation properties amenable to future Smart Radio Architectures.

Within the past few years, wavelet transforms and filter banks have received considerable attention in the technical literature, prompting applications in a variety of disciplines including applied mathematics, speech and image processing and compression, medical imaging, geophysics, signal processing, and information theory. More recently, several researchers in the field of communications have developed theoretical foundations for applications of wavelets as well. Part one of this report surveys the connections of wavelets and filter banks to communication theory and summarizes current research efforts.

This is followed by a discussion of work related to the mathematical development of a flexible, time-frequency-diverse waveform suitable for implementation on modern Smart Radios. Utilizing multidimensional signaling techniques, a generalized multirate wavelet-based modulation format for orthogonally multiplexed communication systems is presented. Wavelet Packet Modulation (WPM) employs the basis functions from an arbitrary pruning of a dyadic tree structured filter bank as orthogonal pulse shapes for conventional Quadrature Amplitude Modulation (QAM) symbols. This generalized framework affords an entire library of basis sets with increased flexibility in time-frequency partitioning. The bandwidth efficiency and power spectral density figures of merit for the general signal are

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derived and it is shown that the power spectral density for every case is equivalently that of standard QAM and hence directly applicable in existing systems employing this modulation format.

Current transceiver designs for wavelet-based communication systems are typically reliant on analog waveform synthesis, however, digital processing is an important part of the eventual success of these techniques. An important part of this effort is a transceiver implementation for the WPM scheme which moves the analog processing as far as possible toward the antenna. The transceiver is based on the Discrete Wavelet Packet Transform (DWPT) which incorporates level and node parameters for generalized computation of wavelet packets. In this transform no particular structure is imposed on the filter bank save dyadic branching, and a maximum level specified apriori and dependent mainly on speed and/or cost considerations. The transmitter/receiver structure takes a binary sequence as input and, based on the desired time-frequency partitioning, processes the signal through demultiplexing, synthesis, analysis, multiplexing and data determination completely in the digital domain - with the exception of conversion in and out of the analog domain for transmission.

A most important application of wavelet packet modulation is in direct sequence spread spectrum (DSSS) communications, where the chip symbols determined by the pseudo noise (PN) sequence are constructed with both time and frequency dimensionality. In traditional DSPN, the wide bandwidth of the symbols causes problems in the presence of frequency-domain noise where all chip symbols are corrupted, but the time-frequency dimensionality of spread spectrum wavelet packet modulation (SSWPM) has immediate advantages in mitigating the effects of narrowband jammers and time impulses, where only a fraction of the chip symbols are corrupted.

The following report details the work accomplished for this project on the above topics – a survey of relevant work, including the investigator's, wavelet packet modulation, the discrete implementation of WPM, and the spread spectrum extension of WPM. These together form a strong body of basic research results which can be carried forth into an

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applied research program focusing on a prototype hardware implementation. At that stage, some of the unanswered questions can be addressed; namely synchronization, symbol timing, and acquisition of the highly orthogonality-dependent wavelet-based waveform. In addition, the multitudinous practical tangents like multiple access applications and embedded forward error correction can be explored.

2 INTRODUCTION

Research in electrical communications and signal processing historically utilizes the best and most recent advances in other fields, mathematics being the most drawn upon field since it provides the analytical framework for proof. Here, it is the fertile field of wavelets and filter banks which is called upon to provide novel approaches to communication problems; and it is a relatively recently developed field at that, having only recently been introduced around 1982 (Morlet, et. al.) The hybridization of perfect reconstruction subband filtering and transmultiplexers by Mallat, Smith, Barnwell, Vetterli, and others, along with the laudable comprehensive treatment given by Daubechies in 1992 has produced a well developed toolbox for the communications researcher. In particular, the time-frequency nature of the wavelet constructions (wavelets, wavelet packets, M-band wavelets, multiwavelets) is appealing, as are their very efficient implementations. The seamless transition from time to frequency dimensionality with a variety of choices in between, is one attraction. The ability of wavelet derived functions to more closely represent a larger class of signals is another attraction where time-frequency capabilities are crucial. From a purely mathematical analysis perspective, the wavelet theory provides a wonderful framework wherein all manner of stylish, and in some strange way artistic, relationships are utilized. Capitalizing on the construction becomes an enjoyable task in this setting, and motivates the researcher to explore. Consequently numerous articles have appeared, most within the last five years, treating communications problems from the multiresolution perspective.

The body of the section presents the overview discussions and is divided into three main parts. The first part is a discussion of waveform design results, which is subdivided into coding, modulation, spread spectrum, multiple access and covert systems categories, reflecting the main themes of articles attacking the different objectives of this problem. Wavelet based channel coding methods, modulation techniques, multiple access (MA) applications, and spread spectrum and covert waveforms are discussed here. The second part is a discussion of interference mitigation results, categorized by transform domain processing and adaptive filtering. Section 2.3 deals with a few other aspects of the communication problem in which wavelet theory has shown promise. Synchronization,

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which many of the results in waveform design assume, is addressed, as well as the modulation identification problem. Detection with wavelets and channel ID (system ID) via wavelets are also discussed here. In addition, an extensive (though certainly not exhaustive) reference section is included, listing the major contributions. From this source, one can trace the vast majority of literature tying communications to wavelets and filter banks.

2.1 Waveform Design

Fundamentally, what makes wavelet theory especially useful in communication systems is the broader class of signals available as "primitive" functions. Traditional Fourier bases, i.e. sines, cosines and their composites, comprise a very small class of functions constrained by periodicity on the interval $[0, 2\pi]$. On the other hand, any $L^2(R)$ (square integrable on the infinite interval) function is a candidate basis function in the wavelet setting. The admissibility condition is certainly a strong constraint, but the real beauty is in the choice. Wavelets of all shapes and sizes have been put forth, each having certain properties useful for a particular application and superior to sinusoidal bases in many applications.

In communication, the properties most called for are the ability to distinguish between desired and undesired signal components and the flexibility of adaptively improving some aspect of the signal representation. Frequency resolution is traded off for time resolution in a seamless and methodical manner. In addition, orthogonality across both scale and translation makes wavelets interesting to the communication community. Systems typically suffer from performance degradation due to intersymbol interference, where the overlapping of adjacent symbols in a given transmission (due to the spreading caused by spectral shaping to conserve bandwidth) causes complications at the receiver. The orthogonality of wavelets and scaling functions rectifies this problem, usually with the caveat that the modulation is done coherently.

From the waveform design standpoint there are several major objectives that arise when applying wavelets to communications. First there is the channel coding of the bit stream, introducing redundancy so as to facilitate error correction at the receiver. Modulation is probably the most active area, where wavelets are in one way or another utilized for symbol transmission. Spread spectrum receives a respectable amount of

attention, where wavelets become a time-frequency spreading vehicle improving on traditional frequency-only schemes. Multiple access is a very active field of application capitalizing on the orthogonality properties and covert system design for military applications is also astir, though most of this work is classified. All of these areas are discussed in this section under separate headings.

2.1.1   Coding

In many communication systems, channel coding is employed as a means to enhance reliability of data transmission across an arbitrary channel. Using the orthogonality properties associated with the wavelet coefficient matrix (WCM), whose rows correspond to the underlying wavelet basis functions, Tzannes and Tzannes [60],[61] have developed a new method of channel coding designed for data transmission over burst noise and fading channels. To code the input signal, k information bits are mapped to n-bit codewords corresponding to the rows of the WCM. Since each row of the WCM is orthogonal to itself and all other rows over appropriate shifts, codewords corresponding to successive k-bit input sequences can be shifted and summed while maintaining their orthogonality with respect to one another. At the receiver, the original data sequence can be decoded using a bank of correlators matched to the codewords in the WCM. In comparison to traditional Hadamard codes, this technique yields a lower probability of symbol error over burst noise and fading channels.

2.1.2   Modulation

Based on the literature, it would seem that modulation is the most popular area of interest to people merging wavelets with communications, as the size of this section attests. Of the current research, about half is geared toward this area. Specific channel environments are generally considered for a given work, usually narrowband or impulsive noise or both. The more general fractal noise processes of the $1/f$ type (which include gaussian noise) are also considered.

Recent efforts utilizing wavelet theory for minimizing the effects of narrowband noise processes have been put forth by a number of people. Wornell and Oppenheim's paper [65] appears to be one of the earliest to suggest the idea of using wavelets for data transmission with a modulation scheme that is very well grounded in theory and analysis.

Starting with a tutorial on deterministically self-similar signals (invariant under scale within an amplitude factor, called *homogeneous*) and then generalizing the family to *dyhomogeneous* functions[1] the connection to wavelets is established and a paradigm for data transmission is formed. The application of the theory is provided in a new waveform design algorithm introduced as fractal modulation, where the term fractal simply refers to the iterative process used to generate the waveform. The optimal receiver for this modulation is a matched filter (which is actually an analysis bank corresponding to the synthesis bank used for modulation) followed by a likelihood ratio test on the samples, which are assumed to be synchronized. However, even though the technique capitalizes on efficient computational implementations it is still a block oriented structure, and so, as the authors point out, it suffers from a significant buffering problem, especially for longer sequences.

In a related work [62], dated slightly earlier, the same process is used to optimize communication over fractal channels, where the additive noise is not necessarily white, but can be characterized more generally by a fractal model, i.e., various degrees of invariance under scale. These random processes are typically referred to as $1/f$ processes and are observed in a number of settings, including optical systems, turbulence flow models, underwater acoustics, electrical systems, natural topology, etc. This work proceeds to represent this fractal noise process with orthonormal wavelet bases and then addresses the problem of bit-by-bit signaling in this environment. With the optimal receiver for that case in place, a multirate modulation strategy for multiple-bit symbols naturally falls out. Ultimately, though the work is done with the gaussian fractal noise environment in mind, it is generally applicable to a much larger class of noise scenarios.

The work just described was thought interesting and practical enough to be analyzed and simulated by Ptasinski and Fellman [49], bringing a certain degree of credibility to the idea of multi-rate modulation with wavelets. Two important figures of merit - power spectral density (PSD) and probability of bit error - are analytically derived for quadrature fractal modulation (QFM,) and it is observed that the PSD for QFM compares favorably with minimum shift keying (MSK) and offset quadrature phase shift keying (OQPSK.) The

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[1] signals that need only satisfy the self-similarity property for dyadic scales, i.e., $x(t) = 2^{kH} x(2^k t)$, for all integers k and constant H, termed the *degree* of the signal.

bit error probability is precisely the same as QPSK. It is suggested here that not only is practical implementation of the fractal modulation scheme feasible (while maintaining near-theoretical performance for the AWGN channel,) but that it also achieves a reasonable improvement over several existing modulation methods.

Cochran [3],[6] suggests "coding" digital communication signals via wavelets. Gandhi [19] also showed that wavelets are useful for analog representation (continuous time waveforms) of data bits, where a conventional Binary Phase Shift Keyed (BPSK) system was modified with wavelet pulse shapes for the bits. This work showed an improvement in bandwidth efficiency[2] over BPSK for all wavelets considered, with almost double improvement for the Battle-Lemarie wavelets. The extension from BPSK to Quadrature Phase Shift Keying (QPSK) was accomplished in [20] where the utilization of orthogonal pulse shapes on both the in-phase and quadrature channels at baseband combined with the well established orthogonality of the radio frequency carriers provides an effective bandwidth efficiency increase of 3½ times that of BPSK.

In an independent effort, Jones' [28] also establishes the very important analytical connection between wavelets and communication signals, paving the way for several future works. Actually, this work could be viewed as the next step in the progression from BPSK to QPSK and then on to M-ary QAM. Starting with a QAM symbol source, a transmultiplexer based on the ordinary wavelet transform is developed which provides time-frequency dimensionality for the transmitted symbols via wavelet pulse shaping. The modulation scheme is called Multi-scale Modulation (MSM) indicating the wavelet scale connection. However, the primary concern here is not bandwidth efficiency, but improved performance in the presence of joint impulsive and narrowband interference. In this framework, these anomalies become less of an influence on system performance since only data symbols in the affected time and/or frequency bands are corrupted. Further efforts [29] enhance the MSM scheme and introduce another wavelet related modulation called M-band Wavelet Modulation (MWM). Incidentally chapter 5 of that work proves a direct connection between Meyer's parametric wavelet construction and the spectral raised cosine nyquist pulse shaping filter. This is perhaps the most unique and worthwhile result, since

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[2] Bandwidth efficiency is defined as the bit rate per unit bandwidth, measured in $\frac{bits/sec}{Hz}$, the system is capable of achieving when the ratio of bit energy to noise power spectral density E_b/N_o , and bit error rate remain constant.

the square root raised cosine is one of the most implemented pulse shapes in use today, with a parametric tradeoff between time and frequency support and linear phase (symmetry) to boot. As far as the authors know, there currently exists no better wavelet for this purpose.

With the multirate modulation idea in place, it is natural then to extend the above MSM and MWM schemes to the more general case of wavelet packets [31] where the structure of the filter bank becomes flexible. In this way, a much broader library of bases upon which to impose M-ary data symbols is available with essentially arbitrary time-frequency tilings. If the block length is chosen to be 2^N then the "supersymbols" contain as many cells, each cell containing the information for one symbol. The net effect is enhanced performance in certain types of channel environments - most notably those with joint narrowband and time-impulsive interferences, in which the narrowband tones are isolated to a few symbols as are the time domain impulses. An algorithm that adaptively selects the optimal set of WPM basis functions for this environment is outlined in [32]. This proposed supersymbol tuning is based on a signal to noise ratio (SNR) objective function, which is desired to be maximized. At each stage in an iterative process the new SNR is computed (measured in a real implementation) and a decision is made as to decomposition or composition of the basis functions. If the SNR is improves then the algorithm progresses, but if not then no further processing is pursued in that direction. The decomposition is accomplished via a growing of children in the filter bank node that corresponds to the basis function in question, and a composition takes two children and prunes them to the parent. Ultimately the supersymbol tuning algorithm arrives at the best wavelet packet basis with respect to SNR.

The optimal supersymbol actually represents a dyadic filter bank of hierarchical quadrature mirror filter pairs and down samplers which accomplishes the transform implementation digitally. It is important to note that the algorithm produces a potentially different tiling for the supersymbol for each block of 2^N symbols. The correspondence between the filter bank outputs and the associated frequency support is shown to follow a gray-code relationship as opposed to a direct mapping because of the inherent aliasing in the downsamplers. In fact, the entire transceiver implementation, including the necessary analysis and explanation of certain nuances which arise, is outlined in [34]. All the results for bandwidth efficiency and spectral density established in [29] still apply but are proved

for this more general setting. As a matter of fact, it is shown in [33] that no matter what wavelet packet decomposition is chosen, the power spectral density is the same for the combined waveform - being equivalent to that of standard quadrature amplitude modulation (QAM).

Wavelet Packet Modulation (WPM) is then a generalized framework for developing communications transceivers with a variety of goals. The inherent noise immunity afforded by the time-frequency dimensionality gives rise to reliability-based or anti-jam (AJ) waveforms, and the potential for using the wavelet packet library as a code space admits low probability of intercept (LPI) waveforms. In addition, multiple access is directly available via the transmitter's demultiplexing and inverse transform stages (potentially each channel of the filter bank can support a single user with complete orthogonality in the combined waveforms.) Finally, since WPM signals are just decompositions of QAM signals, the scheme has an immediate port into existing systems utilizing QAM, with the added benefit of time-frequency dimensionality. Similar work suggesting the use of wavelets for modulation is presented by Erdol in [17]. The work of Wu [66] provides yet another source for results on wavelet packet based modulation, termed wavelet packet division multiplexing (WPDM,) only in this work some effort has been spent investigating the synchronization issue. In addition, multidimensional modulation schemes combined with trellis coding are proposed in [26].

A more practical problem of high performance communication networks is addressed in [53] where a form of multicarrier modulation called overlapped discrete multitone or discrete wavelet multitone (DWMT) modulation. For this scheme, which is based on the application of M-band wavelets, the pulses for different data symbols overlap in time, and are designed to achieve a combination of subchannel spectral containment and bandwidth efficiency that together establish DWMT as superior to other multicarrier modulation techniques. Ultimately the DWMT strategy is geared toward robust performance in the presence of interchannel and narrowband disturbances.

2.1.2.1 Multipurpose Modulation Techniques

The references specifically outlined above are those which are primarily devoted to the idea of applying wavelets and/or filter banks for modulation in conventional single user

communication systems. However, most if not all of them can be directly ported to the multiple access environment and/or the spread spectrum - LPI/D environment. There are several authors whose work straddles the two and so is difficult to classify. For instance, Daneshgaran [8], proposes coherent frequency hopped code division multiple access (FH-CDMA) via the use of scaling functions and wavelets as envelope functions for modulation and ISI-free orthogonally multiplexed transmission over the AWGN channel, and then immediately makes the connection to multiple access, capitalizing on the fact that the frequency supports of M-band wavelets and/or wavelet packets have overlapping frequency support. The wavelet orthogonal frequency division multiplexing (WOFDM) scheme is a modulation format and so deserves mention in this section, but it is intended for multiple access. The same authors in [12] combine WOFDM with frequency hopping and an interesting channel coding scheme rooted in the time-frequency partition defining the wavelet-based waveform (and an additional assumption of coherent demodulation) to get something they call FH/S-CDMA. It is natural at that point for the work to continue the idea into the LPI arena, which they do.

Orr et al [46] combine modulation with other things - in this case spread spectrum and LPI/D. A "new" class of communication systems called wavelet transform domain (WTD) systems is introduced, essentially naming the family of wavelet related systems based on synthesis/analysis (transmultiplexer) implementations. All the potential applications, LPI, covert, bandwidth efficient, multiple access, spread spectrum, are mentioned with the concentration centered around spread spectrum. Though essentially an overview article, a significant amount of useful information is available. Other works which combine the modulation ideas with other features are discussed at greater length in the following sections, mainly because the thrust of the papers is more clearly devoted to a particular waveform design classification.

2.1.3 Spread Spectrum and Covert Communications

Spread spectrum systems are those which utilize spectral spreading techniques to essentially bury the signal in the ambient noise across a very wide bandwidth and then through correlation with the spreading codes, despread the signal at the receiver while the noise undergoes the opposite effect. And though not all spread spectrum waveforms are candidates for covert communication, the converse must be true - spread spectrum is an

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integral and required part of a covert communication system. Hence, many of the articles which deal with spread spectrum from a wavelet standpoint will also venture into the covert/LPI arena. This section attempts to outline the contemporary works which apply wavelets with either goal. For a purely mathematical perspective the work by Benedetto [2] provides a theoretical foundation for secure communication. For another more comprehensive treatment of the connection between spread spectrum and filter banks, the reader is referred to [54] where much of the terminology and methodology of wavelets, filter banks, and spread spectrum communication systems are explained in greater detail.

In addition to detailing MSM and MWM modulation results discussed above, [29] provides a practical extension of the MSM and MWM schemes to spread spectrum, where the MWM waveform is shown to be highly resistant to frequency domain narrowband noise and the MSM waveform is billed as a good compromise between conventional DSPN spreading and MWM when time domain impulsive jammers are also present in the channel. This work provides a comfortable framework in which to analyze orthogonally multiplexed communication systems and establishes a reasonable performance analysis method, if not somewhat constrained by assumption, for the spread spectrum systems which are amenable to that framework. The jump from standard wavelet and M-band decompositions is generalized in [34] to wavelet packets and the spread spectrum extension to this work is available in [35].

The work of Hetling et al [22] uses perfect reconstruction quadrature mirror filters (PR-QMF) for spread spectrum communications. A design framework is presented in [23] which allows for optimization of PR-QMF's subject to spread spectrum system constraints, which are determined by system requirements when operating in a narrowband noise environment. Using the waveform synthesis methodology, LPI/D waveforms with spectral characteristics similar to those of the ambient noise can be generated and used as spreading codes in direct sequence spread spectrum systems. By appropriately defining the constraints and objective functions reflecting the desired signal properties, two-channel PR-QMF banks can be optimized such that their recursive application yields a spreading code which sufficiently resembles the channel environment to render it virtually undetectable to unintended receivers.

Cochran's work on scale based techniques [3],[5],[6] which is discussed in more detail in the next section also has a significant amount of spread spectrum and covertness in the concept. Several noteworthy aspects of SDMA are: 1. Inherent security - those who know the filter sequences can demodulate easily, but without that knowledge it becomes difficult. 2. The previous immediately suggests the possibility of wavelet hopping spread spectrum, where the filters (wavelet pulse shapes) are changed at times known to the communicators only. 3. Compactly supported wavelets provide for LPD waveforms. 4. LPI waveforms are generated by wavelets that distribute the message energy across irregular subbands.

Another approach put forth by Orr et al [45] to designing covert waveforms is to use a pseudo-noise (PN) binary sequence to address the WCM of Section 2.1.1. The actual process used to generate covert waveforms using this technique does not lend itself to a brief summarization. As a result, the interested reader is strongly encouraged to consult the article directly. The methodology presented therein leads to the development of a class of waveforms that can be characterized as energy efficient signals with few features that are detectable by conventional detection/intercept receivers. Interestingly, comparisons with QPSK reveal a noticeable improvement in non-detectability using fourth power law detectors. Another paper with similar content [51] is authored by the same team and so will not be expounded upon here. It is worthwhile to note, however that in this paper an actual hardware testbed for demonstrating and implementing the LPI/D nature of this method is discussed. A complete simulation has been developed using a commercially available simulation package. The technology utilized for this covert waveform generation scheme has also been tested for its applicability to CDMA [47] and it is suggested that user capacity on a given channel can be increased.

2.1.4 Multiple Users

The time frequency flexibility afforded in wavelet constructions admits a framework for selecting or refining waveforms at intermediate time-frequency supports [21]. This allows a seamless transition from the time domain to the frequency domain that is well received in a communications setting. Time domain multiple access (TDMA) ideas easily migrate to Frequency division multiple access (FDMA) ideas within this theoretical framework offering a high degree of flexibility in multiple access signals. The self noise

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inherent in multiple access systems, the so-called multiple access interference (MAI), is more effectively controlled via the hierarchical nature of these multirate transforms in comparison to conventional techniques, and the exploitation of this structure affords a new state of the art in multiple access receiver design.

Work applying wavelet modulations to the multiple access problem started with Cochran [3] where the scale-based coding ideas were first suggested to have application to this problem. Later articles [5],[6],[7] actually solidify the idea. A Scale division multiple access (SDMA) protocol is presented which assigns users to different scales of a mother wavelet. Separation of channels is based on scale as opposed to time or frequency, though a time-division multiplexing arrangement provides for more low-rate users on high rate channels. With the integral shift restriction on the Haar wavelet, all users must have synchronized clocks. This motivates the use of N-band wavelets, which do not have the restriction of integral shifts for orthogonality (they are orthogonal at arbitrary shifts).

A class of bandlimited wavelet bases whose frequency domain "bands" do not overlap are constructed, though they do not (nor can they) come from a multiresolution analysis. The advantage of these wavelets is that due to the orthogonality of the waveforms at arbitrary time shifts, *any* phase shift in the waveforms on different channels (scales) is dealt with successfully at the decoder and hence does not introduce MAI. The result is the capability for *asynchronous* SDMA communication - at the cost of intersymbol interference due to non-time-limited wavelets.

Learned [30] attacks the multiple access problem using wavelet packet derived signature waveforms for concurrent users. The wavelet packet "tableau" (a time-frequency tiling) acts as a design table upon which to assign users unique bins determining their "signature" waveform via the wavelet packet synthesis. All users with signatures from the same level are, by definition, orthogonal, but users at different levels in which one is a descendant of the other introduce MAI. A recursive joint detector handles this problem very well by recursively removing correlations until a very good guess at a bit decision can be made. The system is called wavelet packet multiple access. The method utilizes suboptimal joint detection with significant computational savings over optimal joint detection, and at a minimal performance cost. Also featured in this construction is concurrent signature waveform and receiver design as opposed to modern joint detectors which only consider the

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receiver issues; recursive bit estimation via wavelet packet structure; partially correlated wavelet packet derived signatures instead of DSPN signatures.

A conceptually similar strategy is put forth by Daneshgaran in [8],[12] where the idea is again to combine user waveforms via the inverse wavelet packet transform, thereby accomplishing a wavelet orthogonal frequency division multiplexed (WOFDM) transmission waveform for multipoint to point communications. A frequency-hopping extension to the concept is also presented and follows directly. Synchronization is assumed, based on currently reasonable GPS solutions. The key difference in this strategy is that no overpopulation of the channel is allowed (i.e. all users are orthogonal in the wavelet space), whereas in [30] nonorthogonality is permitted but at the cost of a more complicated receiver. In this work the receiver structure is not discussed but it is readily apparent that due to the similarity with wavelet packet modulation [34] the receiver presented there would apply.

Hetling's approach to multiple access appears in [24] where wavelet based spreading codes take the place of the traditional pseudo-noise (PN) sequence. These wavelet based codes are directly related to the basis functions of a particular wavelet transform or its associated filter bank. Since the choice of available bases is much larger than that of PN sequences, better performance is theoretically attainable by choosing a "best basis" suited for the system requirements. Depending on the environment, the requirements change and thus the objective function that serves as the optimization criterion changes. In the case of the similar work outlined in section 2.1.3, these requirements were based on spread spectrum environments. In this case, multiple access is the objective, and as predicted, the simulation results showed an improvement over Gold codes for equivalent demand (number of users.)

Asynchronous CDMA utilizing multirate filter banks is proposed in [58], with a small user cooperation requirement. In this work, linear decorrelating receivers (completely suppressing multiple access interference) are derived, with the introduction of a criterion that guarantees decorrelation and an optimal user extraction solution in the presence of additive noise. Also addressed in this work is synchronization in environments where both multipath and near-far effects are present. It would seem from the author's perspective that this work is a viable solution to the problems of multiple access interference and near-far power inequities.

2.2 Interference Mitigation

Although the entire previous section would apply here, since all the systems mentioned had some form of interference mitigation in mind, there is a large amount of work specifically devoted to utilizing wavelets and filter banks for removing as much of the interference component in a received signal as possible [1]. These efforts are primarily concerned with spread spectrum signals and utilize transform domain processing techniques.

One significant advantage of spread spectrum signaling is that it inherently provides some protection against interference. In fact, any level of interference protection can be obtained by designing the signal with sufficient processing gain. The price for greater protection, however, is an increase in the bandwidth of the transmitted signal for a given data bandwidth. Practical considerations such as transmitter/receiver complexity and available frequency spectrum can serve to limit the reasonably attainable processing gain. As a result, it is beneficial to apply signal processing techniques to augment the processing gain of the spread spectrum signal itself, allowing greater interference protection without an increase in bandwidth. In general, these *interference suppression* techniques discriminate between the desired spread spectrum signal and the interference and work to suppress the interference.

The primary objective of the work presented in this section is to utilize the time and frequency localization afforded by the discrete wavelet transform DWT and multi-rate filter banks to obtain improved receiver performance in the presence of various types of interference. Clearly, the length of the basis functions directly affects both spectral resolution and computational complexity; as a result, the flexibility in the impulse response duration offered by wavelets and filter banks may be used to improve transform domain resolution. Most importantly, subband filter banks and discrete-time approximations to the (DWT) may be implemented in a manner consistent with the *perfect reconstruction* (PR) property, thus making it possible to recover the time domain signal without distortion. Although the following sections attempt to summarize some of the work performed to date using wavelets and filter banks, most of the work described is on-going and the results presented provide only a preliminary indication of the potential benefits to be gained through the application of the techniques.

2.2.1 Transform Domain Excision

When a signal is transformed or mapped to a different "space" and processed, the signal processing is said to have been done in the transform domain, or, in other words, that one is using transform domain processing. Note that this mapping should be unique and unambiguous and that an inverse mapping or transformation, which can return the signal to the time domain, should exist. In communications and radar applications, particularly ones using spread spectrum techniques, transform domain processing can be utilized to suppress undesired interference and, consequently, improve performance. Here, the basic idea is to choose a transform such that the jammer or the undesired signal is nearly an impulse function in the transform domain, while the desired signal is transformed to a waveform that is very "flat" or "orthogonal," with respect to the transformed interference. A simple exciser, that sets the portions of the transform which are jammed to zero, can then remove the interferer without removing a significant amount of desired signal energy. An inverse transform then produces the nearly interference-free desired signal.

Ideally, the interfering signal appears as an impulse function in the transform domain. In practice, however, transforms such as the Fast Fourier Transform (FFT) and short-time Fourier transform necessitate the use of windowing functions to localize the input data in time. These windowing operations result in frequency domain representations characterized by undesired sidelobes with the amount of energy contained therein directly related to the chosen window. Although the use of non-rectangular windows reduces the size of these sidelobes, it requires the processing of overlapping segments of the input signal in order to accurately reconstruct the time domain waveform, thus greatly increasing the computational requirements.

2.2.2 Excision using Wavelet-based transforms and filter banks

One approach to suppressing interference is to approximate the DWT or discrete wavelet packet using a binary subband tree structure consisting of hierarchical stages of two-channel paraunitary quadrature-mirror filter (QMF) banks [29],[36],[44]. The analysis portion of the overall tree structure consists of cascaded two-channel units which may form a full, dyadic or irregular subband tree. Clearly, different mother wavelets and subband filters yield different values for the corresponding QMF bank filter coefficients. The analysis tree produces transform domain coefficients which can be processed by an exciser

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to remove any coefficients which are determined to be primarily interference. After excision is performed, a synthesis filter bank, constructed from complementary two-channel paraunitary QMF synthesis filters may be used to perform the inverse subband transform.

If the interfering signal is narrow-band, the full binary subband tree structure can be quite effective in localizing a significant amount of jammer energy to a small number of transform domain bins [41]. This method of subband decomposition recursively divides the input signal spectrum into separate lowpass and highpass components. Each resulting component is subsequently partitioned into finer resolution lowpass and highpass frequency bands until the desired level of frequency resolution is obtained. As a result, this method of signal analysis yields a uniform partition of the signal's frequency spectrum much like the DFT. However, unlike the DFT, the spectral sidelobes generated by the filter bank structure are dependent on the QMF filters that are used. Thus, depending on the FIR filter coefficients, the sidelobes may not be as large as those produced by non-windowed DFTs and, hence, fewer bins may need to be excised to remove a specific amount of interference energy. Consequently, given that the same number of bins are removed, subband decomposition of the received signal via hierarchical filter bank structures may yield improved receiver performance relative to that achieved using DFT-based techniques.

From a more general perspective, the original proposition of interference suppression via multirate digital filter banks is usually credited to Jones and Jones [27]. In their work, it was demonstrated that DFT-based polyphase filter bank structures could be used as an efficient alternative to traditional analysis/synthesis techniques. In fact, compared to conventional excision schemes utilizing windowed FFTs, the polyphase structures were shown to improve BER performance without significantly increasing computational complexity.

More recent research efforts have focused on the further development and analysis of transform domain excision schemes based on filter banks and subband transforms [42],[52]. In particular, in [42], modulated and extended lapped transforms were implemented using cosine-modulated filter banks and analyzed in terms of BER performance; similar analysis was also presented for excision schemes using conventional block transforms. It was demonstrated therein that with only modest increases in complexity, lapped transforms may be used to suppress narrowband interferers with

significant improvement in BER performance as compared to conventional block transform excision schemes. In [52] Sandberg, et. al. presented a similar analytical evaluation of excision schemes using cosine-modulated filter banks in place of time-weighted DFTs.

2.2.3 Adaptive time-frequency excision

Tazebay and Akansu [56] introduced the adaptive time-frequency (ATF) exciser as a novel transform domain excision algorithm that adaptively generates a signal dependent hierarchical subband tree using either two- or three-band prototype paraunitary FIR filter banks. This approach analyzes the received signal's energy distribution at each node during the hierarchical subband tree's development and determines whether to continue decomposition with additional filter bank stages or terminate the multi-level tree and process the received signal using the resulting filter bank structure. Since the interfering signal may vary in time, this process is repeated for each received data symbol, thus allowing adaptive reconfiguration of the subband filter bank. In the proposed algorithm, a subband node is decomposed only if its transform domain energy compaction measure exceeds the time domain compaction level and a predetermined decision threshold. As a result, unnecessary decomposition is avoided and, thus, transform complexity, in terms of the number of multiplications and additions, is reduced. In addition, since fewer subbands are decomposed, spectral leakage in the transform domain is minimized.

One of the most promising attributes of the proposed ATF excision algorithm is its ability to perform signal dependent subband decomposition. In contrast to the fixed filter bank structures discussed in the previous section, the ATF excision algorithm adaptively reconfigures the hierarchical subband filter bank structure minimizing transform domain partitioning and improving spectral containment of the interfering signal. It presents a robust transformation technique that tailors the subband frequency responses to the spectrum of the received signal. Consequently, the ATF excision algorithm is less sensitive to jammer frequency than fixed block and subband transforms. Unlike fixed transformation techniques, which may be optimally designed for a given input, this adaptive approach generates a signal dependent subband decomposition capable of tracking and suppressing time-varying interferers. This adaptive quality makes the ATF exciser a valuable interference suppression technique under a variety of channel conditions.

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Recent efforts [48],[59] explore an alternative means of incorporating wavelet transforms and time-frequency distributions into the excision process. In these works, adaptive narrowband interference excision is performed using wavelet and time-frequency distribution analyses of the received signal to drive an FIR notch filter. Here, excision is realized with a bandstop filter whose bandwidth, center frequency and stopband attenuation are adaptively updated in accordance with the jammer characteristics. Although only a few wavelet "primitives" have been tested, preliminary results indicate that this approach may converge faster than conventional linear prediction algorithms.

2.2.4 Adaptive Filtering

Although excision-based schemes are essentially adaptive, they are typically constrained to either keeping or removing an entire transform bin (weighting by zero or one). As a result, subsequent bit decisions are generally not based on a true optimization of any particular performance parameter such as output signal-to-interference ratio or BER. A better system could be formed by adaptively weighting each bin of the transform using continuously-variable tap weights which are determined based on maximizing performance.

For a DS spread spectrum receiver, it may be desirable to minimize the difference between the detected symbols and the output of the correlator. A system that does this and makes use of transform domain correlation is developed in [41]. In this system no synthesis or reconstruction filter bank is required. The input signal and the reference PN spreading sequence are both processed by forward transforms and the results are multiplied point-by-point. This multiplication performs the despreading operation. The products are then weighted by the adjustable taps and then summed in order to produce an estimate of the transmitted data bit which is passed to the decision device to produce the recovered symbol. The error signal that is used to drive the adaptive algorithm is the difference between the transmitted and recovered data bits. If either the LMS or RLS adaptive algorithm is used, the tap weights will converge to minimize the error in the mean-squared sense. For initial startup, a preamble, known to the receiver, can be used in place of the data estimates to *train* the system, with a switch back to the estimates taking place after convergence.

While the adaptive system outlined above can produce a set of optimal weights, it has the drawback that it may not be able to react as quickly to changes in the input as the simple exciser. Most adaptive algorithms are either iterative, in which case they require

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time to converge to the optimal solution, or require information about the statistics of the input signal, which the receiver must estimate when the channel has unknown (and changing) properties. Therefore, if the interference is changing rapidly, the algorithm may not be able to track these changes and will be ineffective at suppressing the interference. Although some algorithms are better at tracking changing conditions than others, there is generally a tradeoff between the tracking capability and the misadjustment noise, which is an indication of the variation of the weight values around the optimal values after convergence. In the literature, the works of Erdol and Basbug [14],[15],[16] address the convergence issues associated with self-orthogonalizing wavelet transform domain LMS algorithms and their implementations. In more recent publications [42],[43] the specific relationships between convergence, misadjustment and the subsequent BER performance of direct-sequence spread spectrum receivers operating in the presence of narrowband interference are analyzed.

2.3 Other Communication Problems

Several issues arising in communication research are yet to be addressed and only a small portion of the literature gives them attention. For instance, *symbol synchronization* is a requirement for the vast majority of wavelet-related modulation strategies and although the assumption of some form of symbol synchronization is common, even with modern technological conveniences like the global positioning system, the synchronization issue is not a trivial one. Daneshagaran addresses this issue with some success [11], deriving prefilters for the synchronizer which completely eliminate the self noise inherent in random data streams (pattern dependent jitter) and thereby reducing the timing error.

Ho [25] addresses the important problem of *modulation identification*, where the desire is to determine the type of modulation used in a detected transmission - potentially for a variety of uses but most obviously surveillance. As it turns out, using the wavelet transform on a digitally modulated signal results in a distinctive pattern, at least for the four types of modulation used as test cases in this work. This makes intuitive sense when one contemplates that these signals are cyclostationary and exhibit amplitude, frequency, and/or phase transients which the wavelet representation is especially useful in identifying. The results indicate promise for high-reliability identification of digital modulation types.

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Detection of weak or unknown signals is another issue which arises frequently. A few authors have contributed work in this area including Ehara [13] for weak radar signals, Frisch [18] for unknown transient signals, and Medley [40] for the detection of spread spectrum signals in the presence of high power narrowband jammers. In most of these cases, the use of wavelets improves upon existing methods. Finally, the problem of *channel identification* (or more generally system ID) has been looked at by at least one author [57].

3 WAVELET PACKET MODULATION

For some time in the communications field, multidimensional signals [29] have been successfully applied to the problems of improving implementation, channel exploitation, message reliability, covertness, etc. Recent developments in multiresolution analysis and wavelet bases [36] have spawned a multitude of interesting applications (see [36] for a review of several related to communications,) so the extension of these results to the multidimensional signaling problem is appropriate [30]. At the beginning of the decade, Coifman, Meyer, Wickerhauser and others introduced the notion of wavelet packets [29] which generalized the already well-grounded theory of wavelets and multiresolution analysis . This theory, arising from the natural concept of "filling out" the nonuniform binary tree used in wavelet decompositions, established that any "arbitrary"3 pruning of the full binary tree would indeed give rise to a basis for $l_2(Z)$. One of the obvious consequences of this construction was rapid computation of M-band wavelets [29] (hen $M = 2^k$ for some integer k) which have application in Jones' M-Band Wavelet Modulation (MWM) communication signals [28]. In this work the theory of wavelet packets is employed to develop a modulation scheme called, appropriately enough, wavelet packet modulation, or WPM. This multidimensional scheme is shown to significantly improve communication performance over QAM [63], MultiScale Modulation (MSM) [28], and MWM for certain channel disturbances while maintaining no less than equivalent performance in others, and this due to the fact that the modulations just mentioned are special cases of the generalized WPM scheme. By making available a library of basis sets a much wider selection of time-frequency tilings is admitted yielding a superior match of the transmission waveform with the channel.

3.1 Construction of Wavelet Packet Bases

In the following, the notation $Z_+ = \{0,1,2,...\}$ is used to denote the non-negative integers and $Z_- = \{0,-1,-2,...\}$ is used to denote the non-positive integers. In like manner

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3The word arbitrary is used rather loosely here, referring to a subset of branches that if decomposed to some maximum level for all, a uniform binary tree would result. I.e., this "arbitrary" pruning must be a valid one.

to [[29], a pair of quadrature mirror filters (QMF) of length L, h_k and $g_k = (-1)^k h_{L-1-k}$, are utilized in the following recursive sequence of functions,

$$p_{2n}(x) = \sqrt{2} \sum_{k \in Z} h_k p_n(2x - k)$$
$$p_{2n+1}(x) = \sqrt{2} \sum_{k \in Z} g_k p_n(2x - k) \qquad , n \in Z_+$$

$$(1)$$

which define the set of wavelet packet functions $p_n(x)$ arising from the given QMF pair. The function $p_0(x)$ is the unique fixed point of the first two-scale equation above and is exactly the scaling function ϕ from a MRA, i.e., the function which forms the basis for the subspace denoted V_1 in [36]. Similarly, $p_1(x)$ is the corresponding wavelet function ψ by the second equation. Indeed, MRA and the wavelet transform (and M-band wavelets at the first decomposition, when $M = 2^k$) are special cases of this general construction. The subspaces generated by these basis functions shall be defined as

$$W_j^n = \overline{\left\langle 2^{j/2} p_n(2^j x - k) \right\rangle}, \quad j, k \in Z, \quad n \in Z_+ \qquad (2)$$

where $\langle \bullet \rangle$ indicates closed linear span. From (1), both p_{2n} and p_{2n+1} are expansions in the scaled function $p_n(2x)$, implying that the spaces generated by these two functions are both subspaces of that generated by the parent. That is W_j^{2n} and W_j^{2n+1} lie in W_{j+1}^n. Furthermore, the widely understood orthogonality properties of the wavelet packet functions are sufficient to admit the general decomposition relation,

$$W_{j+1}^n = W_j^{2n} \oplus W_j^{2n+1} \qquad (3)$$

where the symbol $\oplus$ indicates a subspace direct sum. The proof of (3) is provided in [3]. Thus every "parent" space is decomposed into orthogonal "child" subspaces - completely and without redundancy.

It has been shown [8] that the set of wavelet packets, $p_n(x-k), k \in Z, n \in Z_+$ form an orthonormal basis for $L^2(R)$. For the purposes of this discussion, however, a subset of $L^2(R)$ functions are of more practical interest; namely those that can be represented as expansions in the basis set of the scaling space W_0^0. Functions outside this space do not meet one or more of the criterion for practical communication signals, in particular the

bandwidth, which is intimately related to the sampling rate and defines the initial scaling space from the start. The theorem which establishes this arbitrary pruning of the infinite dyadic tree is also credited to Coifman.

Theorem 1

Let $W_0^0 \subset L^2(R)$ be equipped with orthonormal basis $p_0(x-k), k \in Z$. If the collection $\wp = \{(l,n)\}, l \in Z_-, n \in Z_+$ is such that the dyadic intervals $I_{l,n} = \left[2^l n, 2^l(n+1) \right)$ form a disjoint covering of $[0,1)$, then the set of wavelet packets $2^{l/2} p_n(2^l x - k), k \in Z, (l,n) \in \wp$, form a complete orthonormal basis for W_0^0.

This theorem can be interpreted in terms of the direct sum of the subspaces spanned by each family of packets defined by the elements in the partition $\wp$. That is,

$$W_0^0 = \bigoplus_{(l,n) \in \wp} W_l^n \tag{4}$$

(where it is important to emphasize once again that $l \in Z_-$ and $n \in Z_+$.) In particular, this interpretation clearly indicates that if $p_0(x)$ is a valid scaling function in W_0^0 by (4) it can be written as a linear combination of the basis functions for the wavelet packet subspaces. It is precisely this nexus which admits the use of wavelet packets for communications.

3.2 Waveform Development

The issue to be dealt with in this manuscript is the design of a communication signal having "desirable" properties for a given transmission channel. A desirable signal is one which maximizes the information transferred from sender to observer with minimal distortion and in minimal time, where the distortion and time criterion translate into figures of merit for the signal. To accomplish the task described, the signal must stay within the frequency bandwidth allotted, filling it with as much information as possible while maintaining a relatively small number of erroneous info-atoms. A very good way to do this is to look at the time-frequency plane and see where interferences in the channel show up, whereby a variety of techniques which utilize time-frequency methods can be utilized to optimize communication. The focus of this work is to utilize functions with time-frequency flexibility to construct a signal that will minimize the effect of noise in the channel. Where most excision methods start at the receiver and do analysis first, followed by transform

domain modification and then synthesis for reconstruction, the modulation scheme developed here is intended to overcome channel disturbances at the transmitter (assuming some knowledge of the channel apriori,) synthesizing a signal for transmission first, with decomposition at the receiver.

Jones' development of MWM and MSM was a substantial step in this direction, though these two methods are special cases of this more general scheme allowing for a variety of time-frequency representations. For MSM, a high frequency interferer will do more damage than a low frequency interferer due to the longer bandwidths of the basis functions in those subchannels. For MWM, any time-impulsive noise is bad because all the basis functions spread out over an entire super-symbol. Clearly, a signal with a T-F representation that can be specified arbitrarily (see footnote 3) would minimize the effects of a broader class of interference sources.

Recall the standard QAM signal [63],

$$s(t) = \sqrt{\frac{E_s}{T}} \sum_{m=-\infty}^{\infty} a_m \phi\left(\frac{t}{T} - m\right) \tag{5}$$

where E_s is the average T-second symbol energy of a QAM symbol a_m pulse-shaped by ϕ. Now suppose $\phi \in W_0^0$, i.e., the pulse shape is a wavelet scaling function. The W_0^0 space can be decomposed, as per theorem 1, into a finite set of orthogonal subspaces defined by the collection $\wp = \{(l_1, n_1), (l_2, n_2), ..., (l_J, n_J)\}$ as

$$W_0^0 = \bigoplus_{i=1}^{J} W_{n_i}^{l_i} . \tag{6}$$

The QAM signal, rewritten in terms of the basis functions of these decomposed spaces, becomes the multidimensional signal,

$$s_{WPM}(t) = \sum_{i=1}^{J} \sqrt{\frac{2^{l_i} E_s}{T}} \sum_{m=-\infty}^{\infty} \alpha_m^i p_{n_i}\left(\frac{2^{l_i} t}{T} - m\right) \tag{7}$$

where α_m^i are the complex QAM signals on the ith channel of the associated filter bank which has scale 2^{l_i} (i.e. the channel symbol rate is 2^{-l_i}) with a spectrum concentrated at the band[4] determined by n_i Data placed on orthogonal functions arising from a wavelet packet

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[4] The Gray-coding scheme that determines the bin number of the frequency band is rigorously detailed in [34]. It is not a straightforward mapping as is often erroneously assumed.

decomposition of the pulse shape, compels the name **Wavelet Packet Modulation** or WPM. The orthogonality across scales and within fixed scales of the constituent packets insures a zero ISI waveform, which is also free of cross channel interference.

3.3 Dimensionality and Special Partitions

It is very important to understand that the dimensionality, i.e. the number of T-F atoms in a WPM supersymbol, is fixed in advance as 2^N for some N, and has nothing to do with the number of symbols in the QAM constellation in general. The number of channels J upon which to multiplex the data can vary from 1 to 2^N. Of course the original QAM signal, with no decomposition performed means each symbol occupies the entire bandwidth, but with smaller durations. This corresponds to the partition $\wp_{QAM} = (0,0)$.

The partition $\wp_{MSM} = \bigcup_{i=1}^{J-1} (-i,1) \cup (-(J-1),0)$ is the decomposition defining Multiscale Modulation [28], where 2^{J-1} is the number of QAM symbols occupying $J = N+1$ dyadic frequency bands in the T-F diagram. For the same dimensionality, the partition $\wp_{MWM} = \bigcup_{i=1}^{2^N-1} (-N,i)$ is the decomposition defining M-Band Wavelet Modulation [28], where $M = J = 2^N$, i.e. all symbols occur at the super-symbol rate, occupying equivalent bandwidths much smaller than the data bandwidth. Hence these formats are special cases of the more general WPM scheme.

3.4 A Simple Example

Obviously, because of the inherent flexibility of WPM, it will perform better in many types of interference environments, while keeping the utility of the two special cases MSM and MWM. I.e., there may be an environment which is optimally handled with MSM, and in that case, WPM will conform. The advantage of the WPM is demonstrated in environments where other T-F decompositions would be preferred over MSM or MWM. For instance, consider the case where a narrowband jammer is operating in the high frequency portion of the data bandwidth in conjunction with an impulsive time domain

interferer somewhere in the supersymbol duration. Let $N=4$, providing $2^4=16$ T-F atoms in the decomposition. There will be $N+1=5$ dyadic frequency bands in MSM and 16 equal-width bands in MWM, illustrated by the top two blocks in Figure 1. The normal QAM construction is given in the bottom left and a Wavelet Packet construction is shown in the bottom right.

Notice that for MSM, the tone jammer corrupts 8 of the coefficients and the impulse corrupts 5. One of these is affected by both, thus 12 of the 16 coefficients are noisy. The MWM case has all 16 coefficients affected by the impulse and QAM has all 16 affected by the tone. Thus none of these methods is ideal for these noise sources. However, the WPM diagram in the bottom right, which is optimized for this environment, isolates the interferers
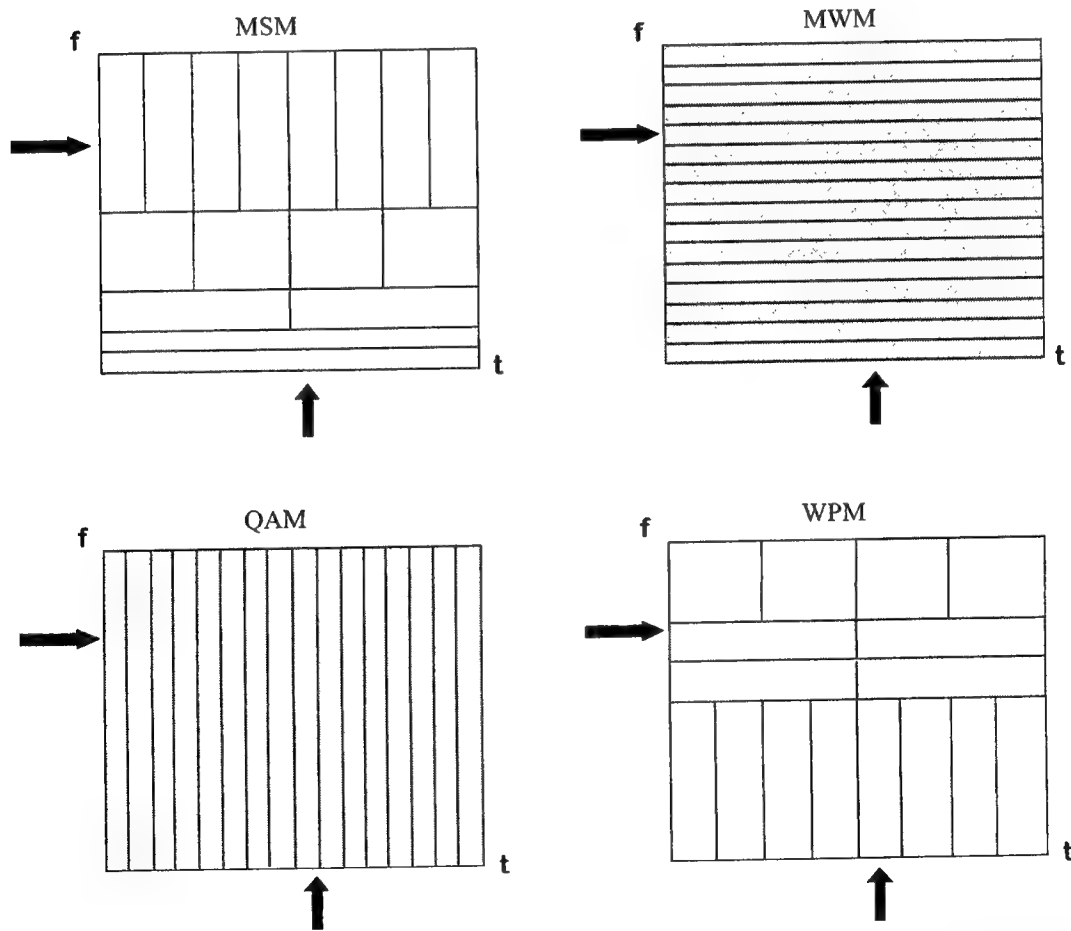

Figure 1.   Comparison of modulation methods for a given interference environment consisting of time impulse and tone jammers.   Shaded areas indicate corrupted symbols.

to 5 of the 16 coefficients - a significant improvement over the previous three. This flexibility to isolate certain channel impairments makes Wavelet Packet Modulation an attractive time-frequency method for multidimensional signaling.

3.5 Waveform Figures of Merit

In order to utilize WPM as a practical signaling strategy, several key properties of the waveform, namely bandwidth requirements (Power Spectral Density), bit error rate, and performance in a Gaussian noise environment, must be quantified. The cases of MSM and MWM are presented in detail in [28], where it was shown that both cases have exactly the same spectral density as conventional QAM. This is really no surprise as all the schemes are variations of the same time-frequency area. However, it is interesting that this property is a function solely of the scaling function and is not dependent on any wavelet function. This is true in the general case of wavelet packets as well. For bandwidth efficiency, the special cases were again shown to be equivalent to QAM, and indeed, since orthogonal pulse shapes are utilized, the results are expected to be consistent. Since the analysis for signals in the presence of noise (bit error rate probabilities) is prohibitive even for relatively simple waveforms, bounds and simulation should be used for quantifying this important signal property. This is not necessarily a problem however, since these are the measures that are used in the actual implementation.

3.5.1 Power Spectral Density
The conventional QAM signal has power spectral density [49]

$$S_{QAM}(f) = E_s |\Phi(fT)|^2 \tag{8}$$

where $\Phi(f)$ is the Fourier Transform of $\phi(t)$, the normalized pulse shaping function, and E_s is the signal energy. In order to derive the power spectral density of the WPM waveform, the following lemma is required:

Lemma: *Let $p_n(x)$ be a wavelet packet function and $P_n(f)$ its corresponding Fourier transform. Then $\forall n \in Z_+$,*

$$\left| P_n\left(\frac{f}{2}\right) \right|^2 = |P_n(f)|^2 + |P_{2n+1}(f)|^2 \tag{9}$$

Proof of Lemma: Taking the Fourier transform of the first equation in (1) gives,

$$P_{2n}(f) = \tfrac{\sqrt{2}}{2} H\!\left(\tfrac{f}{2}\right) P_n\!\left(\tfrac{f}{2}\right) \tag{10}$$

where $H\!\left(\tfrac{f}{2}\right)$ is the half scale Discrete Fourier Transform of the h_k sequence. Squaring the magnitudes of both sides gives

$$\left|P_{2n}(f)\right|^2 = \tfrac{1}{2}\left|H\!\left(\tfrac{f}{2}\right)\right|^2 \left|P_n\!\left(\tfrac{f}{2}\right)\right|^2. \tag{11}$$

By a similar process, the second equation in (1) provides

$$\left|P_{2n+1}(f)\right|^2 = \tfrac{1}{2}\left|G\!\left(\tfrac{f}{2}\right)\right|^2 \left|P_n\!\left(\tfrac{f}{2}\right)\right|^2. \tag{12}$$

so that adding (11) and (12), and applying the well-known [62] "power complementary property", $\left|H\!\left(\tfrac{f}{2}\right)\right|^2 + \left|G\!\left(\tfrac{f}{2}\right)\right|^2 = 2$, gives the desired result. ∎

The PSD of the wavelet packet modulated signal can now be derived. Consider the partition $\wp = \{(l_1, n_1), ..., (l_J, n_J)\}$, and introduce the notation $T_n(f) \triangleq \left|P_n(f)\right|^2$ for convenience. WPM affords the capability of assigning to each channel (basis function) a completely different symbol constellation with possibly varying geometry and/or symbol energies - though they must have the same number of symbols. In this way it is possible for every channel to draw its symbols from different sources (a really attractive feature for coding.) However, the derivation of spectral density in this case is quite involved, and without a closed-form solution. Thus it is assumed that the source is generating independent data and the resulting symbols are then modulated onto J channels (wavelet packets) where the symbols on each channel are identically distributed from the same constellation. The PSD of the WPM waveform is

$$S_{WPM}(f) = E\sum_{i=1}^{J} T_{n_i}(FT2^{l_i}) \tag{13}$$

The proof of this step is exactly the same as that used to show the PSD of ordinary QAM as shown in numerous digital modulation texts, most notably . Now if $\wp$ is sorted to facilitate manipulation such that the ordered pairs follow the rules

1. $l_1 \leq l_2 \leq ... \leq l_J$
2. If $l_i = l_{i+1}$, then $n_i \leq n_{i+1}$, $\tag{14}$

then because of this ordering and the dyadicity of the underlying structure, it must be true that $l_{J-1} = l_J$, and $n_J = n_{J-1} + 1$. Thus (13) can be rewritten as

$$S_{WPM}(f) = ET_{n_{J-1}}(fT2^{l_{J-1}}) + ET_{n_J}(fT2^{l_J}) + E\sum_{i=1}^{J-2} T_{n_i}(fT2^{l_i})$$

$$= ET_{n_{J-1}}(fT2^{l_J}) + ET_{n_{J-1}+1}(fT2^{l_J}) + E\sum_{i=1}^{J-2} T_{n_i}(fT2^{l_i}) \qquad (15)$$

$$= ET_{\underset{2}{n_{J-1}}}(fT2^{l_J-1}) + E\sum_{i=1}^{J-2} T_{n_i}(fT2^{l_i}),$$

where the last line is allowed by the lemma. This relation is actually the PSD of another WPM waveform with partition $\wp' = \{(l_1', n_1'), \dots, (l_{J-1}', n_{J-1}')\}$ where $(l_{J-1}', n_{J-1}') = \left(l_J - 1, \frac{n_{J-1}}{2}\right)$, and this new partition can again be reordered as per (14). This process can be continued $J-1$ times, yielding a final partition $\wp' = \{(0,0)\}$. Thus the expression for the PSD of WPM becomes

$$S_{WPM} = ET_0(fT2^0) = E|P_0(fT)|^2. \qquad (16)$$

Also, recalling that p_0 is just the scaling function from an MRA at the same scale, yields

$$S_{WPM}(f) = E|\Phi(fT)|^2, \qquad (17)$$

which is precisely the power spectral density of the QAM signal. The significance of this result is that every WPM waveform has a bandwidth requirement equivalent to QAM, and may be utilized on channels currently supporting this traditional modulation.

3.5.2 Bandwidth Efficiency

For this figure of merit it is instructive to first consider the general case of separate constellations for each channel. Let the ith channel carry symbols α_m^i from a 2^{B_i}-QAM constellation, where B_i is the number of bits per symbol and R_i is the symbol rate. The PSD in (17) has some bandwidth

$$W_\beta = \frac{1+\beta}{T}, \qquad (18)$$

where T is the period of the pulse-shaping function $\phi(t)$, and β is the percent excess bandwidth required beyond the Nyquist signaling bandwidth. So the general expression for the bandwidth efficiency of channel i is

$$\rho_i = \frac{R_i B_i}{\frac{1+\beta}{T}} = \frac{R_i B_i}{W_\beta}, \qquad (19)$$

and therefore the bandwidth efficiency of WPM is

$$\rho_{WPM} = \sum_{i=1}^{J} \rho_i = \frac{T}{1+\beta} \sum_{i=1}^{J} R_i B_i . \tag{20}$$

Now, from the WPM waveform (7), the period for each symbol on channel i is $2^{-l_i} T$, so the ith channel rate is $R_i = \frac{2^{l_i}}{T}$ and hence (20) becomes

$$\rho_{WPM} = \frac{1}{1+\beta} \sum_{i=1}^{J} 2^{l_i} B_i . \tag{21}$$

This is as much simplification as is possible when considering the general case of different QAM constellations for each channel. However, for the case where $B_i = B$, i.e., all channels have the same symbol length (and therefore the same number of symbols in each constellation, although no constraint has been placed on the geometry,) the bandwidth efficiency becomes

$$\rho_{WPM} = \frac{B}{1+\beta} \tag{22}$$

because the l_i's come from a dyadic partition of the interval $[0,1)$, forcing the constraint $\sum_{i=1}^{J} 2^{l_i} = 1$. QAM using the same pulse shaping filter has precisely the same bandwidth efficiency. Hence, the theoretically expected result: increasing dimensionality by orthogonal decomposition of the pulse shaping function does not improve bandwidth efficiency.

4 DISCRETE IMPLEMENTATION OF WPM

Wavelet Packet Modulation (WPM), a modulation scheme based on wavelet packets, has been developed in detail, but it remains to be seen how this format can be realized in a digital signal processing framework. It is certainly expected that a transceiver similar to that developed by Jones in [29] would accomplish the task, and it turns out that this is indeed the case. However, in order to justify the discrete implementation of WPM which will be developed in the next section, it is necessary that the Discrete Wavelet Packet Transform (DWPT) relative to some partition , and the corresponding inverse operation (IDWPT) be illuminated in sufficient detail. The reader is perhaps familiar with the standard wavelet transform and its associated non-uniform filter bank structure, or the M-Band wavelet (and fixed level wavelet packet) transform and the associated uniform filter bank structure. Presently not so widely understood is the generalized wavelet packet transform and the associated generalized filter bank. It is easy to deduce the heuristic operation of this structure, but a degree of mathematical exactitude is provided for completeness, and in the process, the additional notation needed for further developments is established.

4.1 The Discrete Wavelet Packet Transform and its Inverse

Consider the recursively defined wavelet packet functions[5],

$$p_{2n}(x) = \sqrt{2} \sum_{k \in Z} h_k p_n(2x - k)$$
$$p_{2n+1}(x) = \sqrt{2} \sum_{k \in Z} g_k p_n(2x - k)$$
$$, n \in \{0,1,2,...\} \qquad (23)$$

where the h and g sequences are discrete quadrature mirror filters (QMF) [8]. In the context of the current discussion, these sequences are interpreted as series coefficients for the functions $p_{2n}(x)$ and $p_{2n+1}(x)$ cast in the basis of double-scaled, translated p_n functions. It is these functions which are used to build the WPM signal. In terms of inner products the series coefficients take the form

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[5] The $n = 0$ case of (23) is the familiar two-scale multiresolution analysis equation set where ϕ and ψ take the place of p_0 and p_1 respectively.

$$h(k) = \left\langle p_{2n}(x), \sqrt{2}\, p_n(2x-k) \right\rangle = \sqrt{2} \int_{-\infty}^{\infty} p_{2n}(x) p_n(2x-k) dx$$

$$g(k) = \left\langle p_{2n+1}(x), \sqrt{2}\, p_n(2x-k) \right\rangle = \sqrt{2} \int_{-\infty}^{\infty} p_{2n+1}(x) p_n(2x-k) dx$$

(24)

Now, let W_j^n represent the vector space generated by the wavelet packet functions

$2^{j/2} p_n\left(2^j x - k\right)$, $j, k \in Z$, $n \in Z_+$. Since the packet basis functions are orthonormal,

any function f in this vector space has a plethora of corresponding wavelet packet series',

each associated with a unique "partition"   of the unit interval.   A particular series is

manifest as coefficients $a_l^n(i)$ in

$$f(x) = \sum_{i \in Z} \sum_{(l,n) \in \wp} a_l^n(i) \sqrt{2^l}\, p_n\left(2^l x - i\right).$$

(25)

These coefficients are computed via inner products with the basis functions, i.e.,

$$a_l^n(i) = \left\langle f(x), \sqrt{2^l}\, p_n\left(2^l x - i\right) \right\rangle = \sqrt{2^l} \int_{-\infty}^{\infty} f(x) p_n\left(2^l x - i\right) dx,$$

(26)

and constitute the Discrete Wavelet Packet Transform of f relative to   . The following

'single node decomposition' via the quadrature mirror filters

$$a_l^{2n}(i) = \sum_{k \in Z} h(k-2i) a_{l+1}^n(k) = \left[H a_{l+1}^n \right](i)$$

$$a_l^{2n+1}(i) = \sum_{k \in Z} g(k-2i) a_{l+1}^n(k) = \left[G a_{l+1}^n \right](i)$$

(27)

provides for the efficient calculation of the transform coefficients without the costly inner

product integration,   where H and G represent the operations of filtering by h or g

respectively, followed by decimation by 2.   These relations are proven in numerous

references, e.g.[36].   The signal processing operations for the relations of (27) actually

represent one stage in a wavelet packet decomposition as shown in Figure 2a where the

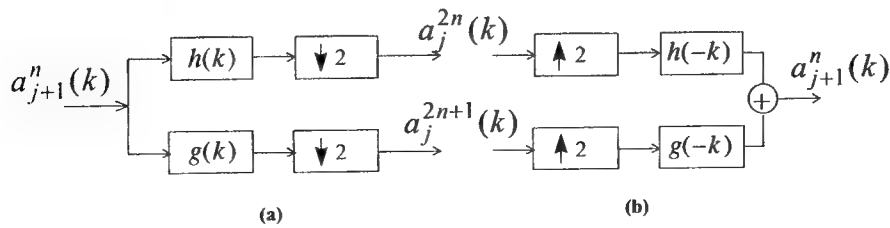signal lines are labeled with the relative coefficient sequences.

Figure 2. Single Stage of the (a) WPT and (b) IWPT

It should be noted that, in this general setting, the complete wavelet packet filter bank defined by the partition has no pre-programmable structure. I.e., the decomposition is not necessarily to some uniform level, nor is it necessarily a wavelet transform. Therefore the actual algorithm which computes the coefficients must be slightly more involved from a bookkeeping perspective, keeping track of when a particular terminating node has been reached, as well as the partition coordinates. If the current node is not one of the partition coordinates, then another stage of decomposition is performed at that node. The generalized composition (synthesis) result is

$$a_{l+1}^{n}(i) = \sum_{k \in Z} h(i - 2k)a_{l}^{2n}(k) + \sum_{k \in Z} g(i - 2k)a_{l}^{2n+1}(k)$$

$$= H * \left[a_{l}^{2n} \right](i) \quad + \quad G * \left[a_{l}^{2n+1} \right](i)$$

(28)

and provides an equally efficient method of calculating the IDWPT for the given partition. This relation represents the signal processing of Figure 2b.

In the communication signal design problem, this latter building block is used to *generate* the WPM signal, and the former is used for reception and detection. This is a twist on the vast majority of the literature for which the concentration is on *signal analysis* and/or *compression* by processing the wavelet coefficients or by thresholding them respectively. After the objectives are accomplished the signal is reconstructed - perfectly in the case of signal analysis and near-perfectly in the case of signal compression. However, in this application, the objective is to "build" a custom signal that exploits the channel properties and avoids non-Gaussian noise if possible. Hence, the operations are reversed.

4.2   The original WPM Transceiver

Now that the requisite concepts are understood, the actual generation of a WPM signal can be considered. For the sake of review, the original definition of WPM is

$$s_{WPM}(t) = \sum_{i=1}^{J} \sqrt{\frac{2^{l_i} E_s}{T}} \sum_{m=-\infty}^{\infty} \alpha_m^i p_{n_i} \left(\frac{2^{l_i} t}{T} - m \right).$$

(29)

where α_m^i represents the complex QAM data indexed by m traversing the ith channel of the filter bank defined by the current partition $\wp = \{ (l_1, n_1), (l_2, n_2), \ldots, (l_J, n_J) \}$. It is entirely plausible that WPM be implemented directly with analog pulse shaping filters for each partition coordinate, requiring a complex scheme for tracking the symbols, and their

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different rates. Additionally the receiver would require matched filters for every pulse shape, which is exponentially significant for large dimensionality. The preferred method, however, is to take advantage of the structure in the inverse wavelet packet transform to construct a signal determined by the QAM symbols. Figure 3 shows how this is accomplished.

The sequence of complex QAM data symbols, $a(k)$, is used as the source for wavelet packet coefficients and after demultiplexing this symbol stream into J channels at appropriate rates, the substreams α^i are applied to the synthesis process of the IDWPT. This forms a sequence of "coded" complex symbols, $b(k)$, which are intimately related to the QAM data. *These new symbols do **not** map to a QAM constellation* but do retain the information for "decoding" via the inverse transform. The symbols become weights for dirac impulses which are then shaped by the scaling function pulse-shaping filter for transmission. At the receiver, a single filter matched to the scaling function followed by a sampler provides the reconstructed symbol estimates, $\hat{y}(k)$, and the analysis process of the DWPT (for the same partition as that used in the synthesis) expands these noisy symbols, resulting in estimates, $\hat{a}(k)$, of the original QAM data.

The process can be shown mathematically starting with the $b(k)$ sequence out of the IDWPT. The signal $y(t)$ is a weighted train of impulses given by

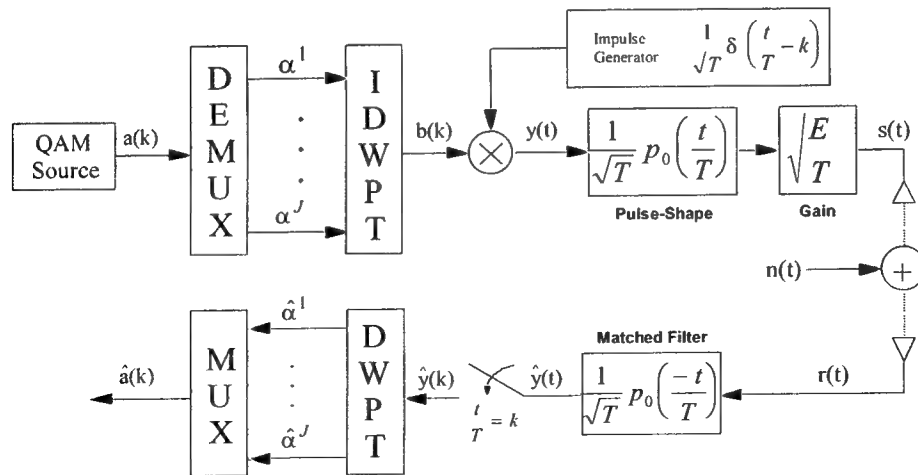$$y(t) = \sum_{k=-\infty}^{\infty} b(k) \frac{1}{\sqrt{T}} \delta\left(\frac{t}{T} - k\right) \tag{30}$$

Figure 3. WPM Transceiver Model

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which after convolution with the pulse-shaping filter and gain factor becomes

$$s(t) = \sqrt{\frac{E}{T}} \int_{-\infty}^{\infty} yp(t-\tau) \frac{1}{\sqrt{T}} p_0\left(\frac{\tau}{T}\right) d\tau$$

$$= \sqrt{\frac{E}{T}} \sum_{k=-\infty}^{\infty} b(k) \int_{-\infty}^{\infty} \frac{1}{\sqrt{T}} \delta\left(\frac{t-\tau}{T}-k\right) \frac{1}{\sqrt{T}} p_0\left(\frac{\tau}{T}\right) d\tau.$$

$$(31)$$

Changing variables as $u = \frac{\tau}{T} \Rightarrow d\tau = Tdu$, gives

$$s(t) = \frac{1}{T} \sqrt{\frac{E}{T}} \sum_{k=-\infty}^{\infty} b(k) \int_{-\infty}^{\infty} \delta\left(\frac{t}{T}-u-k\right) p_0(u) \, T \, du. \tag{32}$$

yielding, by the sifting property of delta functions,

$$s(t) = \sqrt{\frac{E}{T}} \sum_{k=-\infty}^{\infty} b(k) \, p_0\left(\frac{t}{T}-k\right) \tag{33}$$

which looks a great deal like the original QAM signal

$$s(t) = \sqrt{\frac{E}{T}} \sum_{k=-\infty}^{\infty} a(k) \, p_0\left(\frac{t}{T}-k\right) \tag{34}$$

In fact, the only difference in these two signals is the fact that the complex symbols $b(k)$ can take on immensely more values than the 2^k values $a(k)$ of the original QAM symbol set. As a matter of record, the range for $b(k)$ has cardinality

$$N_b = \left(\left(2^k\right)^L\right)^J = 2^{kLJ} \tag{35}$$

where k is the number of bits per symbol in the QAM constellation, L is the length of the QMF filters, and J is the number of input channels in the filter bank. For example, with a BPSK constellation (only two symbols), 2-tap Daubechies QMF's, and a two-channel filter bank, the output sequence can take on any of $2^{1 \cdot 2 \cdot 2} = 16$ values, which is a modest eight-times expansion of the original symbol set. For perspective however, a 16-QAM constellation (comparatively low for modern technology), 37-tap square-root raised cosine QMF's, and an 8-channel bank provide $2^{4 \cdot 37 \cdot 16} = 2^{2368} > 10^{712}$ possible output values - a virtual continuum! This has significance in certain applications where low probability of interception is important, since the symbol constellation is no longer nicely partitioned for easy decoding.

At the receiver, the transmitted signal is corrupted additively by noise, $n(t)$, in the channel so that the output of the matched filter is

$$\hat{y}(t) = \int_{-\infty}^{\infty} s(t-\tau) \frac{1}{\sqrt{T}} p_0\left(\frac{-\tau}{T}\right) d\tau \quad + \quad n_F(t) \tag{36}$$

where $n_F(t)$ is the filtered noise. Substituting (33) and changing variables as above yields

$$\hat{y}(t) = \sqrt{\frac{E}{T}} \sum_{n=-\infty}^{\infty} b(n)\sqrt{T} \int_{-\infty}^{\infty} p_0\left(\frac{t}{T}-u-n\right)\phi(-u) du \quad + \quad n_F(t). \tag{37}$$

Sampling at $t/T = k$,

$$\hat{y}(k) = \sqrt{E} \sum_{n=-\infty}^{\infty} b(n) \int_{-\infty}^{\infty} p_0\left(-u+(k-n)\right) p_0(-u) du \quad + \quad n_F(k) \tag{38}$$

which after incorporation of the orthonormality property of the scaling function at integer shifts gives

$$\hat{y}(k) = \sqrt{E} \sum_{n=-\infty}^{\infty} b(n)\delta(k-n) \quad + \quad n_F(k) \tag{39}$$

$$= \sqrt{E}\, b(k) + n_F(k)$$

which is a sequence of estimates of the original symbols out of the inverse wavelet packet transform. Decoding these estimates is accomplished with a forward wavelet packet transform and demultiplexer, the result being a sequence resembling the original QAM data. Just how well these output symbols represent the original ones is a function of the noise and the partition chosen for the WPM expansion.

4.3 Practical Modifications

The design heretofore presented is fine in theory, but the reader may be concerned with the impracticality of the dirac delta function generator for digital to analog conversion. On this point the author concedes that more practical methods of accomplishing the pulse shaping for bandwidth efficiency must be incorporated for implementation of WPM. This problem was solved, at least for special cases of multi-scale modulation and M-band wavelet modulation, by Jones [29]. It will be shown that this same strategy can be applied to the general case of wavelet packet modulation.

Consider the modulated signal in (33). This expression can be interpreted as a series expansion of $s(t)$ in the basis of translated functions $p_0\left(\frac{t}{T}-\bullet\right)$. In this interpretation, the series coefficients are represented by $b(k)$. Using the relation for p_0 in (23), this becomes

$$s(t) = \sqrt{\tfrac{E}{T}} \sum_{n=-\infty}^{\infty} b(n)\sqrt{2} \sum_{k=-\infty}^{\infty} h(k) p_0\left(\tfrac{2}{T}t - 2n - k\right) \tag{40}$$

Making the variable substitution $l = 2n + k$ gives

$$s(t) = \sqrt{\tfrac{E}{T}} \sum_{n=-\infty}^{\infty} b(n)\sqrt{2} \sum_{l=-\infty}^{\infty} h(l - 2n) p_0\left(\tfrac{2}{T}t - l\right)$$

$$= \sqrt{\tfrac{E}{T}} \sum_{l=-\infty}^{\infty} \left[\sqrt{2} \sum_{n=-\infty}^{\infty} b(n) h(l - 2n) \right] p_0\left(\tfrac{2}{T}t - l\right) \tag{41}$$

where the bracketed expression is the familiar upsample-and-filter interpolation operation shown in Figure 4.

$$b(n) \longrightarrow \boxed{\uparrow 2} \longrightarrow \boxed{h(n)} \longrightarrow$$

Figure 4. Signal processing recursively hidden in WPM transmit signal.

Thus the dirac impulse generator and the analog pulse shaping filter can be replaced by the digital processing above followed by a digital to analog converter and a simple image rejection filter with decent cutoff approximating the scaling function p_0 scaled by two. The end result is a digitally implemented transmitter for wavelet packet modulation.

The receiver portion of the transceiver model also has some analog processing due to the matched filter. Duality arguments allow this block to be replaced by discrete components as well, for it is obvious that reversing the last steps in the transmission process will "undo" that processing. Thus a moderately tight anti-aliasing filter followed by analog to digital conversion will give received samples resembling those at the transmitter prior to digital to analog conversion and filtering for image rejection. Then a deconvolution of the received samples to undo the effect of the final filter stage at the transmitter and downsampling by two provides the input to the inverse wavelet packet transform.

There is however, the matter of the deconvolution, which should raise some flags for the astute reader. It is true that not any filter will suffice for the transmitter pulse shaping stage. One requirement is that the filter have a nonzero discrete Fourier transform. If this sequence has any zero at all, it will be impossible to invert the transform for equalization and the desired deconvolution filter at the receiver will not exist. In fact, even when the filter has a non-zero definite transform, the dynamic range can be a roadblock, since the

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deconvolution sequence will have values with magnitudes in the range of the reciprocals of the pulse shaper sequence. For instance, in the case of a filter with h(1)=.001, the deconvolution filter will have values in the range 1000. This dynamic range problem is an issue that is best addressed by scaling the filter to straddle unity as close as possible and adjusting the gain stage accordingly.

4.4 Translation Between Tiling Diagram and Filter Bank

A tiling diagram is really just a graphical representation of the partition defining the actual basis functions utilized in the signal which generates it. In other words, the partition and the tiling diagram are equivalent ways of describing a signal composed of wavelet packet bases - the difference manifested mainly in the way the description is interpreted. A tiling diagram describes the basis functions in terms of their frequency localization and symbol rate, and the partition describes the basis functions directly as output nodes of a dyadic filter bank whose nodes are labeled as ordered pairs. The goal of this section is to establish a method of translating from one to the other, and in the course of this development, the concept of gray-coding is utilized. At this point it is necessary to review the notation and generation of gray-codes.

4.4.1 Gray Coding of Frequency Bins

Gray encoding is a binary coding scheme which was originally developed to circumvent the effects of large transient errors in electrical counters. Communication systems use the coding method for building in immunity to large bit error in certain signaling systems. The idea behind gray codes is that any symbol is only one bit different from its nearest neighbors in the code. Therefore, since most errors are due to the noise causing the data symbol estimate to stray into a neighboring symbol's detection region, there will only be one bit in error rather than two or more, thus providing lower bit error rates. However, reducing errors is not the only application of gray codes. It turns out that many situations which involving the use of binary notation and/or dyadic structure are ripe for a gray code application. In the case of dyadic perfect-reconstruction filter banks, the structure of the banks admits a natural application.

Consider the sequence of sets, G_k, in which first three elements in the sequence are

$$G_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad G_2 = \begin{bmatrix} 00 \\ 01 \\ 11 \\ 10 \end{bmatrix} \qquad G_3 = \begin{bmatrix} 000 \\ 001 \\ 011 \\ 010 \\ 110 \\ 111 \\ 101 \\ 100 \end{bmatrix} \tag{42}$$

These three sets are the first three terms in the more general recursion

$$G_k \triangleq \begin{bmatrix} G_k(0) \\ G_k(1) \\ \cdots \\ G_k\left(2^k - 1\right) \end{bmatrix} = \begin{bmatrix} 0G_{k-1}(0) \\ 0G_{k-1}(1) \\ 0G_{k-1}\left(2^{k-1} - 1\right) \\ 1G_{k-1}\left(2^{k-1} - 1\right) \\ \\ 1G_{k-1}(0) \end{bmatrix} = \left[\begin{array}{c|c} \bar{0} & G_{k-1} \\ \hline \bar{1} & G_{k-1}^{flip} \end{array} \right]. \tag{43}$$

where G_{k-1}^{flip} is the "upside down" version of G_{k-1}. The set G_k is actually a bijective

mapping from the set $\{0, \cdots, 2^k - 1\}$ to the set of k-bit binary sequences. In this light, G_k

has an inverse given by

$$G_k^{-1}\left(d_{base2}\right) \triangleq n \quad \ni \quad G_k(n) = d_{base2} \tag{44}$$

which follows from the definition.

Now, the Paley or natural ordering for the labels of the filter bank nodes has already

been chosen. At level k, the nodes are labeled in order from top to bottom, starting at 0 and

proceeding through $2^k - 1$. If the data bandwidth is divided into 2^k subintervals and the nth

bin is labeled (starting at 0) with the decimal equivalent of $G_k(n)$, a direct mapping from

the filter bank node n at level k to the nth frequency bin results. Figure 5 illustrates this

important connection.

4.4.2 Translation Computations

Given the above information it is a simple task to translate a coordinate pair in some

partition to its frequency bin in the bandwidth. To be very specific, if N is the maximum

level of the wavelet packet filter bank (MAXLEVELS) and B and T are the data bandwidth and QAM symbol duration respectively, then other important information such as bin width and symbol rate for the channel can be found. Clearly the supersymbol duration is $T_{ss} = 2^N T$. Now consider the partition coordinate (l, n). For this coordinate the bandwidth is divided into 2^l subintervals and the filter bank nodes are labeled according to the Gray coding procedure above. The packet described by this coordinate has the following frequency parameters

$$\text{index} = G_l^{-1}\left(n_{\text{base 2}}\right) \triangleq b$$
$$\text{width} = \frac{B}{2^l} \tag{45}$$
$$\text{center freq.} = \frac{2b+1}{2^{l+1}} B$$

The time parameters of the T-F atoms generated at this node of the filter bank are

$$\text{width} = \frac{T_{ss}}{2^{N-l}} = 2^l T \quad \Rightarrow \quad \text{cell rate} = \frac{1}{2^l T} \tag{46}$$

To complete the idea, consider a specific partition coordinate, say (-3,7), of a filter bank with $N = 4$. This coordinate is the last node at level 3 in the filter bank, and describes a set of T-F atoms with the following parameters

$$b = G_3^{-1}\left(7_{\text{base 2}}\right) = G_3^{-1}\left(111\right) = 5$$
$$\text{width} = \frac{B}{2^3} = \frac{B}{8}$$
$$\text{center freq.} = \frac{2(5)+1}{2^{3+1}} B = \frac{11}{16} B \tag{47}$$
$$\text{rate} = \frac{2^{4-3}}{T_{ss}} = \frac{2}{T_{ss}}$$

The translation from frequency bin to wavelet packet number is simple: The decimal representation of the gray code $G_k(b)$ for the bin is the kth level wavelet packet number. Or more precisely,

$$\left[G_k(b)\right]_{base10} = n. \tag{48}$$

The scale or level number l is directly associated with the atom-rate for the channel and the value of N, as seen from (46) so that

$$l = N - \log_2\left(rate \cdot T_{ss}\right) \tag{49}$$

Thus a systematic method of moving from the tiling diagram to the filter bank (i.e. basis functions), and vice-versa, has been established. It is interesting to note that if the operators H and G are substituted for 0 and 1 respectively, in $G_l(b)$, the resulting sequence, call it $F_l(b)$, is exactly the filter sequence necessary to obtain frequency localization in bin b at level l.

4.5 Why Gray-coding?

The answer to this excellent question is revealed through an important property of the decimation blocks in the filter bank[6]. If h_k and g_k are discrete lowpass and highpass quadrature mirror filters with bandwidths $\left[0, \dfrac{F_s}{4}\right)$ and $\left[\dfrac{F_s}{4}, \dfrac{F_s}{2}\right)$ for a given sampling rate F_s, then the decimation blocks cause a certain exploitable form of aliasing in which the lowpass and highpass bands of the output of a g-filter and decimator actually switch
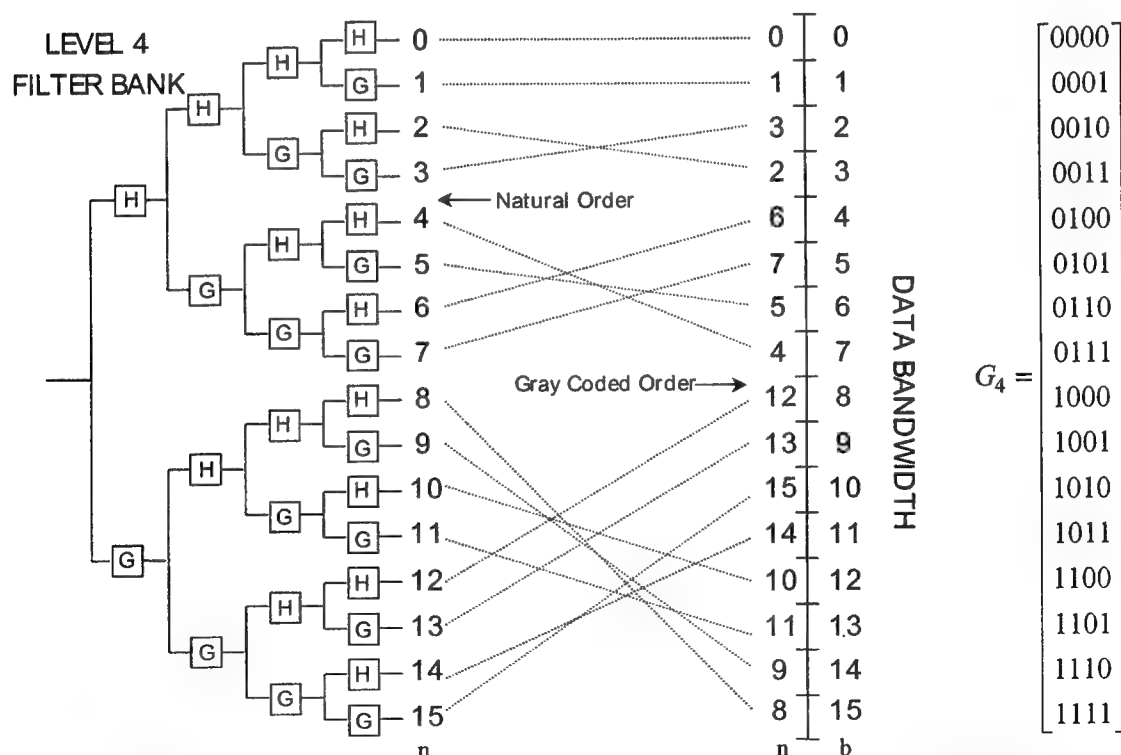

Figure 5. Connection between Paley or natural ordering of filter bank nodes and sequency or Gray-coded ordering of frequency bins.

[6] Jones calls it "band-shuffling."

positions relative to the original bandwidth. Thus two consecutive G operations filter a signal to the bandpass region $\left[\dfrac{F_s}{4}, \dfrac{3F_s}{8}\right)$ and a GH operation filters the same signal to the highpass band $\left[\dfrac{3F_s}{8}, \dfrac{F_s}{2}\right)$. The proof of this interesting phenomenon is provided in [9]. Since this only occurs at the output of the G-filters, and since the complete filter bank is made up of iterated two-channel filter banks, the gray coded frequency localization comes naturally.

5 EXTENSION OF WPM TO SPREAD SPECTRUM

In this work, the time-frequency selective processing features inherent in wavelet packet modulation are extended to spread spectrum waveforms where redundancy created by large dimensionality is exploited at the receiver for interference mitigation. Actually, the work should probably be viewed as a generalized extension to that of Jones [28]. (It is noted here that efficient implementations are established for the system proposed, but these are not the subject of this work.) The most attractive feature of this spread spectrum wavelet packet modulation (SSWPM) system is the time-frequency flexibility which incorporates several methods as special cases. Indeed, DSPN, which has only time dimensionality (each data symbol waveform is decomposed in time with short "chips" utilizing the entire data bandwidth) is one special case, as is spread spectrum M-band wavelet modulation (SSMWM) which has only frequency dimensionality in a given symbol (each data symbol waveform is composed of separated narrowband pulses covering the entire symbol period). Other hybrid systems exist, all of which are special cases of this general SSWPM scheme.

Notions recently introduced in support of the development of standard WPM [34] are useful. In particular the "supersymbol" and its associated "tiling diagram" (borrowed from multiresolution analysis theory) provide graphical justification for the unified approach to spread spectrum communications. From these diagrams it is plain to the reader how dimensionality in time or frequency or both are theoretically perceived. The viewpoint taken also allows for the wavelet packet modulation receiver to be viewed as a projection operator, where the wavelet packet basis functions defining the transmitted waveform are the target of the projection. The resulting projection coefficients can be optimally weighted for maximum received signal to noise ratio and hence minimum bit error rate.

For SSMSM and SSMWM, these optimum weights have been shown to exhibit predictable behavior depending on the character of the interference. In the case of widespread influence where the interference is basically uniform across the supersymbol, a uniform weighting is shown to be most appropriate, whereas localized interference is best handled with excision of the corrupted coefficients. This latter case arises when an impulsive jammer is met by the appropriately partitioned supersymbol, for instance when a dirac impulse corrupts DSPN or a narrowband jammer corrupts SSMWM or multitone

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modulation. In fact, the SSMSM signal which uses a standard wavelet basis for the pulse shapes and hence partitions the supersymbol in the very familiar dyadically related rectangles was shown to be a very good compromise for joint time and frequency jammers or when the character of the jammer was unknown but assumed impulsive in either domain.

5.1 Signal development

Traditional DSPN waveforms have the following form

$$s(t) = \sqrt{\frac{E}{T}} \sum_{i=0}^{\infty} \alpha_i \sum_{j=0}^{M-1} c_j \varphi \left(M \left(\frac{t}{T} - i \right) - j \right) \tag{50}$$

where the α_i's are binary data, and the c_j's are binary "chips" from an M-chip sequence shaped by the function φ which has energy E within the symbol duration T (this function is usually a square pulse.) From the work done on Wavelet Packet Modulation, it was established that asserting a MRA scaling function as the pulse shape provided significant benefits for system performance. In addition, since the scaling functions are shifted in time, the DSPN signal in (50) has complete dimensionality in time.

Similarly, a spread spectrum signal having frequency domain dimensionality has the form

$$s(t) = \sqrt{\frac{E}{MT}} \sum_{i=0}^{\infty} \alpha_i \sum_{n=0}^{M-1} c_n \psi_n \left(\frac{t}{T} - i \right) \tag{51}$$

where the ψ_n's are (perhaps overlapping) pulse shaping functions each having (perhaps overlapping and/or orthogonal) spectral occupancy significantly smaller than the data bandwidth but which when taken together cover it completely. Since each of these functions has duration T, the symbol period, the signal has complete dimensionality in frequency.

It is natural then, to pursue a trade-off between these two signals, having dimensionality in both time and frequency. A first step in this direction was taken with the Spread Spectrum Multiscale Modulation (SSMSM) signal, also by Jones. Unfortunately, this signal also suffers from a lack of flexibility, and the performance in the presence of narrowband interference varies widely depending on the frequency of the jammer. It is natural then to apply Wavelet Packet Modulation as a generalization of these cases where a hybrid time-frequency dimensionality is the goal. The SSWPM signal then takes the form

$$s(t) = \sum_{i=0}^{\infty} \alpha_i \sum_{j=1}^{J} \sqrt{E_j'} \sum_{k=0}^{2^{N+l_j}-1} c_k' p_{n_j} \left(2^{N+l_j} \left(\frac{t}{T} - i \right) - k \right) \qquad (52)$$

where $E_j' = \frac{2^j E}{T}$, the energy in the packet function associated with the jth frequency bin in the data bandwidth, and $c_k' = c_{(2^{N+l_j} n_j)+k}$, the complicated subscript on the chip variable arising from the subtleties involved in applying a coding sequence in a multirate system - in this case a filter bank with J output channels and a maximum of N levels. The pulse shapes p_{n_j} are wavelet packets defined by the (l_j, n_j) coordinate of the wavelet packet partition [34].

In Wavelet Packet Modulation, the waveform is generated via the decomposition of a QAM source [63] at some symbol rate into several sub-rate QAM signals followed by a synthesis procedure with a perfect reconstruction filter bank whose structure depends on the desired time-frequency partitioning of the transmitted signal. The spread spectrum modification to this signal involves the intelligent application of a pseudonoise spreading code to the pulse shaping functions carrying the information, just as in all direct sequence spread spectrum waveforms.

5.2 Implementation

The implementation of the SSWPM transceiver follows very similarly to that used for standard wavelet packet modulation [34], with the inclusion of the PN code application. First a bank of J multiplexers with a total of M inputs all receive the same input data symbol. Each multiplexer block also acts as a PN code applicator, so that each input is modulated by one chip of the M-chip sequence (which may come from a longer code.) The multiplexer outputs are then at the correct rate for each input on the inverse wavelet packet transform. The reader will notice that the only difference between the resulting signal and standard WPM is the symbol rate - WPM operates at M times the symbol rate of SSWPM.

The intelligent multiplexing mentioned above is a consequence of the multirate nature of the filter bank. Specifically, the inputs to the synthesis bank in the transmitter operate at potentially different rates, depending on decomposition level. Larger levels (higher negative numbers) require lower rate input sequences, and vice versa. To account for this feature, a multiplexer dependent on the partition elements is incorporated.

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At the receiver, the noisy input signal is processed by a filter matched to the wavelet packet scaling function pulse shape used to construct the transmitted signal, then the output is sampled at the appropriate time (timing and symbol synchronization are assumed, though this issue is not at all trivial.) The resulting real or complex (depending on whether the quadrature channel is also used in transmission or not) symbol sequence is transformed via the discrete wavelet packet transform defined by the partition used in the transmission. The output channels of this transform, just as in the transmitter's inverse transform, carry sequences whose rates depend on the level of decomposition for each channel. Again, this multirate character must be accounted for intelligently, hence the presence of demultiplexers which provide to the summer the appropriate M values from J channels every M^{th} sample instant. Figure 6 illustrates the process

It should be noted that the demultiplexers in this diagram do more than just split out sequences into single values. Every output channel contains a multiplier that reapplies the corresponding PN chip applied at the transmitter, resulting in PN removal. The output of the summer then becomes the transmitted symbol.
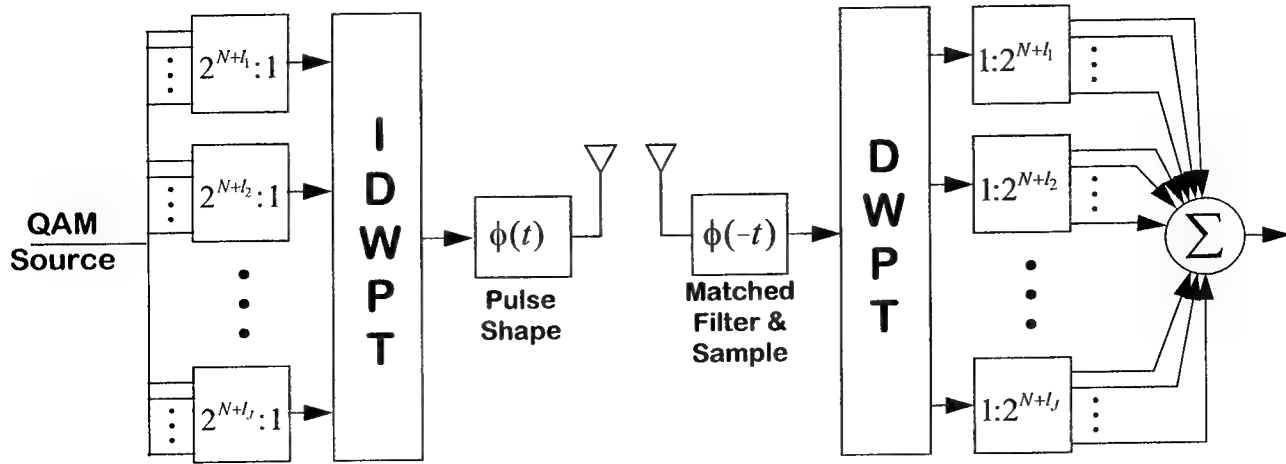

Figure 6. Implementation of SSWPM - note similarity to standard Wavelet Packet Modulation

5.3 Impulsive Noise Localization

As mentioned above, the advantage of SSWPM is realized most clearly in the presence of joint time and frequency jammers. Figure 7 shows an example of the optimized supersymbol obtained for the jammer scenario consisting of a single tone and single dirac

impulse. The gray-shaded areas represent those "chips" in the symbol which are corrupted directly by the non-gaussian noise sources. In this case there are 128 chips per symbol and only eight are contaminated by the noise. In an excision system [36], where the Grey shaded areas are identified and canceled before reconstructing the signal, this tight localization would translate into much higher ratio of received signal to jammer power.

In comparison, Figure 8 shows the fixed TF tiling of a SSMSM signal. Since the tone is in the lower half of the data bandwidth, SSMSM performs very poorly with well over half of the supersymbol (spread spectrum symbol) corrupted. The characteristic of straight SSMSM is the standard wavelet basis used for constructing the signal. This basis has a high rate lowpass component that occupies the entire lower half of the data bandwidth. This feature is detrimental when narrowband jammers operate in that range, corrupting half of the supersymbol without any help from a time domain impulse. In this case, of 128 chips, 71 are a liability to the received SNR. Clearly, the flexibility afforded by the wavelet packet construction is useful for mitigating the effects of these types of jammers.

5.4 Performance

Figure 9 shows the BER for the four signals mentioned in section 1 - DSPN, SSMWM, SSMSM and SSWPM, as a function of signal to jammer energy[7] (SJR.) Clearly, the spread spectrum Wavelet Packet Modulated waveform with an optimized supersymbol
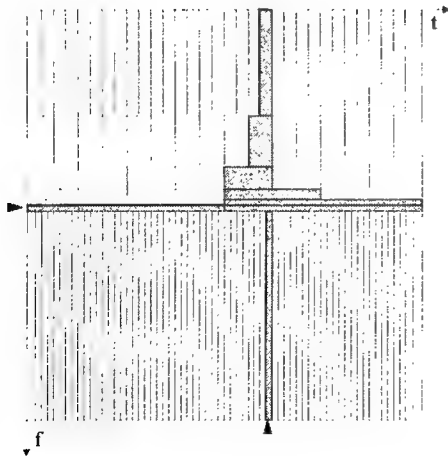

Figure 7. Example of interference localization in a 128-cell supersymbol.
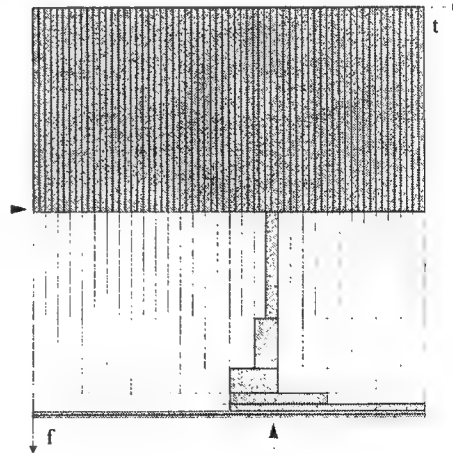

Figure 8. Example of interference localization for the SSMSM signal in a 128 cell supersymbol.

[7] Since the jammer is on for the entire duration, i.e. 100% duty cycle, energy and power ratios are equivalent.

tiling performs dramatically better in the aforementioned joint impulsive jammer scenario. The graph does require some explanation however. In this example, which does not correspond to the previous two figures (they were deliberately less complicated due to space and resolution limitations,) 1024 chips were applied to each transmitted symbol, and the ambient noise power was set to 9.6 dB. For all four modulation types (all special cases of the SSWPM waveform), in the absence of any jammer energy, the bit error rate for simple AWGN at 9.6dB SNR is expected to be exactly the same as the well known binary phase shift keyed (BPSK) system - approximately 1 error every 100,000 bits. This is the "noise floor" for high SJR.

Now, at very low SJR, jammer energy dominates, so the asymptotic behavior of the curves can be readily derived. It is straightforward to show that (based on the particular jamming scenario of one tone and one dirac impulse in the supersymbol,)

$$\lim_{SJR \to -\infty} BER = \frac{X/M}{2} \tag{53}$$

where X is the number of corrupted chips and M is the total number of chips per symbol. For DSPN (high rate blocks occupying the entire data bandwidth) and SSMWM (blocks occupying the entire supersymbol duration but only a fraction of the data bandwidth,) the
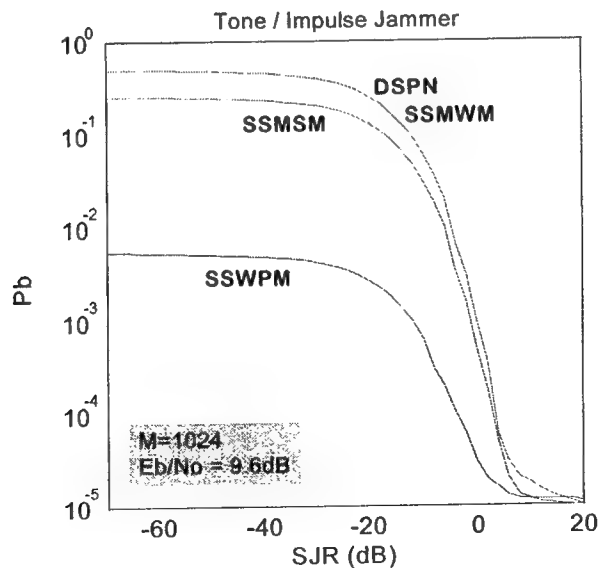

Figure 9. BER Comparisons for DSPN, SSMWM, SSMSM, and SSWPM for a single tone/impulse jammer.

entire supersymbol is corrupted either by the tone or the dirac pulse. Thus the jammer will dominate the signal at low SJR and therefore the probability of correctly detecting a bit is 1 in 2 – flipping a coin would work as well. For SSMSM, with the tone in the lower half of the bandwidth, and the dirac impulse anywhere in the supersymbol duration, there are $X=512+\log_2 1024=522$ chips corrupted. Hence the asymptotic BER is around .255. For optimized SSWPM (again, optimized for this particular jamming scenario) which has a predictable corrupted chip count of $1+\log_2 M=11$, the asymptotic BER is about .0054, explaining the markedly improved performance in low SJR regions.

Finally, it should be noted that the relative performance theoretically increases ad infinitum. That is, as the number of chips per symbol rises, the asymptotic BER decreases proportionally. However, a trade-off with frequency localization spreading due to increased filter bank decomposition keeps this performance increase within practical limits.

6 CONCLUSIONS

6.1 Literature

The application of wavelets and filter banks to communication systems has received a great deal of attention in the last 5 years, with many of the communication objectives, channel coding, modulation, spread spectrum and covertness, multiple access, interference mitigation, synchronization, detection, channel ID and modulation identification being addressed in particular articles. Though there is a significant amount of overlap among researchers in this field, it is encouraging to see the amount of activity and the promising results being developed. In addition, from the reference sections of many papers discussed in this summary article, it is easy to see a spirit of collaboration and sharing of information as well.

6.2 Wavelet Packet Modulation

The results presented here have established at least three important ideas. First, the connections between the mathematics of Coifman and Wickerhauser's wavelet packet constructions and Quadrature Amplitude Modulation via the decomposition of a scaling function pulse shape are openly revealed. Secondly, the bandwidth efficiency and power spectral density for the WPM signal are derived from first principles, showing the equivalence of the subject signaling strategy to ordinary QAM and hence portability into existing systems. Thirdly, the application of interference mitigation via flexible time-frequency tilings afforded by the generalized wavelet packet construction is attacked for joint time-frequency impulsive jammers and a significant improvement in SNR is shown to be obtainable.

The work is not without its unsolved problems, however. In any communication system, the practical issues of acquisition and synchronization are fundamental to implementation. Since the goal of this work was to establish mathematical connections between wavelet packets and communications - thereby capitalizing on the programmability and efficiency of the digital filter bank for implementation, the assumptions of accurate timing are useful for the development, but they still must be addressed at some point for the WPM strategy to be utilized. The acquisition part is not

difficult, since the PSD for the WPM system is equivalent to QAM, it is certainly plausible that a standard carrier tracking device would suffice. Symbol synchronization is the real problem, since the wavelet packet pulse shapes vary in scale (dilation). As it turns out, every communications signal that conveys information (non-degenerate signals) inherently possesses extractable features - one in particular is the nondifferentiability of the transmitted signal at certain points where the data cycles between bits. Minimum Shift Keyed (MSK) waveforms attempt to reduce this effect by smoothing the transitions, but a good feature detector will still reveal them. The WPM strategy is no different in this respect, since the data transitions generate "edges" in the transmitted signal which can be extracted for timing purposes. Work is currently being done to solve the timing problem by capitalizing on the edge-detection or discontinuity "alarm" capabilities of wavelets - potentially combining the synchronization and detection processes for increased computational efficiency.

The transceiver for this waveform is very interesting, though space restrictions prohibit the detailed discussion here. Essentially, the waveform construction contained in this report affords a very efficient *all digital* filter bank transceiver incorporating a programmable demultiplexer followed by a synthesis filter bank which performs an inverse wavelet packet transform on the demultiplexed data streams. The resulting "coded" data then passes through a single pulse shaping filter before transmission. At the receiver, a matched filter and sampler (this is where the symbol timing becomes crucial) provide a data stream that is input to an analysis filter bank which mirrors that of the transmitter, performing a discrete wavelet packet transform on the input sequence. The output channels are intelligently multiplexed to produce an estimate of the original data sequence.

6.3 Digital Implementation of WPM

The main focus here is to optimize the performance and increase the implementability of this signaling scheme in order to engender practical development of transceivers which utilize the technology, and benefit from its jammer-avoidance properties as well as its channel-fitting adaptability in both the time and frequency domains. In the pursuit of this goal the discrete wavelet packet transform was reviewed and its application to WPM solidified. Specifically the difference between an analysis-first scheme that decomposes an incoming signal for analysis and processing, then synthesis for reconstruction and a

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synthesis-first scheme (transmultiplexer) that composes a signal from data for transmission and then analysis for breaking out the data is addressed. After presenting the mathematical make-up of the WPM signal and its dependence on analog processing for pulse shaping, a modification for the transmitter was presented which replaces the continuous-time dirac function generator with a digital equivalent, capitalizing on the signal processing recursion hidden in the WPM structure. A dual modification for the receiver using an anti-aliasing filter followed by A/D conversion comprises the replacement for the analog matched filter. The problem of deconvolution of the transmitter filtering and associated problems with dynamic range are acknowledged – and have been addressed in a cursory fashion. In addition, the issue of frequency bin allocation and indexing was described and the problem of translating wavelet packet number to/from frequency bin solved analytically. The result is a completed work on the basic block-diagram-level implementation for Wavelet Packet Modulation.

6.4 Spread Spectrum Wavelet Packet Modulation

The SSWPM results are not surprising, for "matching" the signal to the channel characteristics intuitively means better performance. However complexity becomes an issue, since a full binary tree is very computationally intense depending on the order of the FIRs and the channel bandwidth. A practical feature of SSMSM is that the dyadic tree is linear in the number of FIRs, therefore trading performance for complexity. Robustness is another issue, since the performance degrades substantially with a mismatched waveform. The transmitter must have knowledge of the channel characteristics and, some mechanism must exist to dynamically adjust the transmitted signal. This problem has been dealt with in [32] but the solution requires either a reliable feedback channel from the receiver to the transmitter which will adjust the transmit waveform on a dynamic basis depending on channel conditions measured by the receiver (this has some obvious practical limitations) or the transmitter must sense the channel. These issues make for some interesting future research.

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